Optimal Selective Forwarding for Energy Saving in Wireless Sensor Networks

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***Abstract*—Scenarios where nodes have limited energy and forward messages of different importances (priorities) are frequent in the context of wireless sensor networks. Tailored to those scenarios, this paper relies on stochastic tools to develop selective message forwarding schemes. The schemes will depend on parameters such as the available battery at the node, the energy cost of retransmitting a message, or the importance of messages. The forwarding schemes are designed for three different cases: 1) when sensors maximize the importance of their own transmitted messages; 2) when sensors maximize the importance of messages that have been successfully retransmitted by at least one of its neighbors; and 3) when sensors maximize the importance of messages that successfully arrive to the sink. More sophisticated schemes will achieve better importance performance, but will also require information from other sensors. The results contribute to identify the variables that, when made available to other nodes, have a greater impact on the overall network performance. Suboptimal schemes that rely on local estimation algorithms and entail reduced computational cost are also designed.**

***Index Terms*—Sensor networks, wireless sensor networks, energy-aware systems, Markov Decision Process.**

I. Introduction

ENERGY consumption is a primary concern in Wireless Sensor Networks (WSN). This is because in many practical scenarios, sensor node batteries cannot be (easily) refilled, and nodes have a finite lifetime. Since every task carried out by WSN has an impact in terms of energy consumption, many solutions have been proposed in the literature to optimize energy management; see, e.g., [3], [4].

Communication processes are typically among the most energy-expensive of such tasks. Despite the fact that a proper design of the physical layer can contribute to reduce communication costs in WSN, see, e.g., [3], [5], energy savings can also be obtained by taking a higher level approach and considering the different nature of the information that nodes have to transmit. In order to enlarge the network lifetime and optimize the overall network performance, sensor nodes could weigh up: a) the potential benefits of transmitting information and b) the cost of the subsequent communication process. A first step to address such an optimum design is to properly quantify or estimate both costs and benefits. This is possible in practice because the energy consumed by every communication task (cost) is typically well-characterized and because applications where messages are scored according to an importance indicator (benefit) are frequent in WSN. The message importance can be, for instance, a priority value established by the routing protocol, or an information value specified by the application supported by the network. Some relevant examples can be found in the fields of security (attack reports [6]), medical care (critical alerts [7]), or data fusion (DAIDA algorithm in [8]), to name a few. Once costs and benefits are properly quantified, energy can be saved by making intelligent importance-driven decisions about message transmission.

The idea of selective communications, which is the basis of this paper, consists of discarding low importance messages in order to save energy that can be used to transmit more important messages in the future. In order to make a decision, sensors will take into account factors such as the energy consumed during the different node states, the available battery, the importance of the received message, the statistical distribution of such importances, or the behavior of their neighbors. Related ideas have been explored in the literature, either using a heuristic approach (typically focused on the modification of existing algorithms; see, e.g., the IDEALS algorithm in [9]) or a theoretical one (aiming at the identification of basic guidelines for WSN design). Theoretical approaches base the design of importance-driven schemes on the resolution of a specific optimization problem. That is the case for the decentralized detection algorithm proposed in [10], where every sensor decides if the observation is transmitted to the fusion center based on the information carried by its observation and the energy consumption. Another relevant example is the work in [11], where a mathematical framework based on Markov chains is used to characterize an optimal policy for single hop transmission over a replenishable sensor network.

Also starting from theoretical considerations, in [1] we formulate the design of selective communication policies as a stochastic sequential decision problem, proposing a mathematical model that is a particular case of a Markov Decision Process (MDP) [12]. The application of MDP models to

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sequential decision in WSN has attracted recent attention. It has been used as a tool to find a trade-off between the energy savings of data aggregation and the transmission delay [13]; to balance the energy saving of low-power sensor states and the efficiency of the sensing, receiving and transmitting processes [14]; or to optimize a reward function combining power consumption, throughput and delay [15]. Our approach in [1] is content-driven: the importance is used to decide whether transmit or discard a message so that the accumulated importance of all transmitted messages is maximized.

The model in [1] offers powerful insights and guidelines for the design of schemes able to exploit the trade-off between message importance and energy consumption. However, it does not allow the forwarding policy of a given node to depend on parameters of other nodes. Generalizing the model to allow the use of information from other nodes, and analyzing the impact of using non-local information in the network behavior are the main goals of this paper. To do so, we develop optimum forwarding schemes for three different scenarios: 1) when sensors maximize the importance of *their own* transmitted messages (which is similar to the setup in [1], but using a more general energy consumption model); 2) when sensors maximize the importance of their messages that are actually *retransmitted* by their neighbors; and 3) when sensors maximize the importance of the messages that successfully *arrive to the sink*. Upon properly selecting the formulation, all three cases can be tackled in parallel. Interestingly, the experimental analysis shows that the use of local information from neighboring nodes (second scenario), which requires a minimal signaling overhead, outperforms the first scenario, and it is very close to the third one, which would entail higher signaling cost. This suggests that the whole network behavior can be optimized by using only local information.

Regarding practical implementations, it will turn out that the optimal forwarding scheme is fairly simple: the decision maker must compare the importance of the received message with a time-variant threshold. Furthermore, under some stationarity conditions a constant threshold (which can be estimated adaptively) is nearly optimum. The results in this paper generalize those in [1] because two additional scenarios (optimality criteria) are considered and the assumptions here are less restrictive. Especially important is the generalization of the results to stochastic energy costs. Moreover, the adaptive schemes are more robust and require less a priori information than those in [1].

From a generalization point of view, although the schemes in this paper solve well-defined problems and are derived using a self-contained formulation, they can be adapted to address problems different than those specifically considered here. For example, our approach can be easily integrated with a variety of existing data collection approaches, as in-network data aggregation. On the other hand, although the paper has a strong theoretical component, the results are also useful from a practical point of view. Because not only can they provide basic guidelines for the design of future systems, but also the developed schemes can eventually be incorporated into many existing routing protocols.

The rest of the paper is organized as follows: Section II describes the context and the model for the WSN. The optimization problem is solved in Section III. Assuming stationarity in the importance distributions and large batteries, an asymptotically optimum scheme that gives rise to a constant threshold that does not vary along time is also developed. Adaptive methods to estimate the forwarding thresholds are proposed in Section IV. Theoretical results will be complemented with numerical simulations in Section V. Conclusions in Section VI wrap-up this paper.

II. Sensor Model

A. State vector

For the purpose of the analysis that follows, we consider a sensor network as a collection of nodes $\mathcal{N} = \{n | n = 0, \ldots, N-1\}$. For the time being, we will focus on the behavior of each node, that receives a sequence of requests to transmit different messages (no matter how the network topology is). The node state will be characterized by at least two variables

- e_k: available energy (battery level) at time k. It reflects the "internal state" of the node; and
- x_k: importance of the message to be sent at time k. It reflects the "external input" to the node.

Besides e_k and x_k, the node can use additional information to make decisions: this includes information about the packet (e.g., the packet length, which could be relevant to estimate the energy cost of forwarding it) and some data about the state or the eventual actions of neighboring nodes (e.g., information about the forwarding policy of other nodes). All this additional information (together with x_k) is collected into vector $\mathbf{z}_k$. Following a usual terminology in MDP models, the state vector of the node is defined as $\mathbf{s}_k = (e_k, \mathbf{z}_k)$; i.e., the state vector contains all and only the information that is available at the node to make a decision at time k. The set of all possible states is denoted as $\mathcal{S}$.

The "time" variable k requires some explanation. Strictly speaking, it does not represent physical time, but a counter of epochs. An epoch is every time period that starts when the node gets some data (from the sensing devices or from other nodes) with a request to forward them to the sink node, ends when the node discards the message or completes the transmission to a neighboring node, and contains all time instants devoted to receive, process and (eventually) forward these data. In practice, this may happen in a non-continuous time period (e.g., if new messages arrive to the node before completing the transmission of previous messages), and different epochs may be intertwined in the real time line. The identification of the time periods that correspond to each epoch (and the further assignment of energy consumption to epochs) is an important step for implementing a selective forwarder in true networks, but goes beyond the scope of this paper.

In addition to this, it may happen that a node has no pending messages, and stays in idle state or listening to the channel without any transmit/discard decision to make. For our mathematical model, these periods can be interpreted as requests to transmit null messages with zero importance.

B. Actions and policies

At time k, the sensor node must make a decision d_k about sending or not the current message. The message is sent if

$d_k = 1$, while it is discarded if $d_k = 0$. A forwarding policy $\pi = \{d_1, d_2, \ldots\}$ at a given node is a sequence of decision rules, which are functions of the state vector; i.e.,

$$d_k = d_k(\mathbf{s}_k) = d_k(e_k, \mathbf{z}_k). \tag{1}$$

Note that, with some abuse of notation, we adopt the same notation for the decision variable and the decision function.

C. State dynamics

Nodes consume energy at each time epoch by an amount that depends on the taken actions. We will express the available energy at time k recursively as

$$e_{k+1} = e_k - d_k c_{1,k} - (1 - d_k) c_{0,k}, \tag{2}$$

where $c_{1,k}$ is the energy consumed when the node decides to transmit the message, and $c_{0,k}$ is the energy consumed when the message is discarded. The latter may include the cost of sensing the data (if the sensor device is the source of the message), the cost of data reception (when data come from other nodes) or the cost of idle states (if there are no data to transmit which, as stated before, is equivalent to receive a virtual zero importance message). Parameter $c_{1,k}$ accounts for all the previous costs plus the cost of forwarding the message. In general, we assume that energy consumption may depend on $\mathbf{z}_k$ and may have some random components, so that $c_{1,k}$ and $c_{0,k}$ are stochastic processes. This way we can deal with a broader range of scenarios. One example are WSN transmitting over fading channels, where the energy consumption is a random variable that depends on the fading realization; see, e.g., [5]. Another example is a WSN where errors induce packet losses and nodes implement automatic repeat request (ARQ) schemes to combat them. In those networks, the energy required for a successful transmission can vary with a certain probability due to the additional cost that retransmissions entail. With respect to the other component of the state vector, i.e., $\mathbf{z}_k$; we assume that it is a statistically independent sequence, and independent of e_{k-n} or d_{k-n}, for any $n > 0$.

Our goal is to use the previous assumptions to characterize the probability of any state transition from k to $k+1$. More specifically, we want to find $p(\mathbf{s}_{k+1}|\mathbf{s}_k, d_k)$, which denotes the probability of reaching the state s_{k+1} given that at time k the state was s_k and the decision made was d_k. To do so, let $p_{0,k}$, $p_{1,k}$ and p_{k+1} denote the probability density functions of $c_{0,k}$, $c_{1,k}$ and $\mathbf{z}_{k+1}$, respectively. Taking into account that the energy consumed during time $k+1$ is $e_k - e_{k+1}$ and that the energy dynamics is given by (2), the transition probability $p(\mathbf{s}_{k+1}|\mathbf{s}_k, d_k)$ can be expressed as

$$\begin{aligned} p(\mathbf{s}_{k+1}|\mathbf{s}_k, d_k) = (&d_k p_{1,k}(e_k - e_{k+1}|\mathbf{z}_{k+1}) \\ &+ (1 - d_k) p_{0,k}(e_k - e_{k+1}|\mathbf{z}_{k+1})) p_{k+1}(\mathbf{z}_{k+1}). \end{aligned} \tag{3}$$

In other words, if $d_k = 1$, then $p_{1,k}(e_k - e_{k+1}|\mathbf{z}_{k+1}) p_{k+1}(\mathbf{z}_{k+1})$; while if $d_k = 0$, then $p_{0,k}(e_k - e_{k+1}|\mathbf{z}_{k+1}) p_{k+1}(\mathbf{z}_{k+1})$. Although for the theoretical analysis we assume that the distributions $p_{0,k}$, $p_{1,k}$ and p_{k+1} are known, we will see that only some of its statistics, which may be estimated from data, are required to implement the selective forwarder.

D. Rewards

Let $q_k \in \{0, 1\}$ denote the *success index* (a binary variable taking value 1 if the transmission is successful, and zero otherwise). With $u(\cdot)$ standing for the Heaviside step function (with the convention $u(0) = 1$), the *reward* at time k for a node that decides to transmit a message will be

$$r_k = x_k q_k u(e_k - c_{1,k}). \tag{4}$$

The "success" of a transmission can be measured in different ways. In this paper we consider three different measures:

- *Global success index*. Since each message must travel through several nodes before arriving to destination, the message transmission is completely successful ($q_k = 1$) if the message arrives to the sink node, and zero otherwise.
- *Local success index*. If the transmitting node has no way to know if the message arrives to the sink, the global success index is not accessible. However, it may be the case that the transmitting node can know if the neighboring node receiving the message forwards it to other nodes or not (by listening to the channel, or because the neighboring node returns a confirmation message). The local success index is $q_k = 1$ if a neighboring node forwards the message, and zero otherwise.
- *Zero-order success index*. As a degenerate case, we can take any transmission as successful, so that $q_k = 1$ in any case. This amounts to say that every node maximizes the importance of its own transmitted messages, which is the problem investigated in [1].

In summary, if d_k denotes the decision at node i, (4) states that the node receives a reward equal to the message importance if $d_k = 1$ (i.e., the node decides to transmit the message), $q_k = 1$ (i.e., the transmission is successful), and $e_k \geq c_{1,k}$ (i.e., the node has enough energy for the transmission). Otherwise, the reward is zero. In all three previous cases we have assumed that when a node transmits a message, the message is always received by the destination. This free-loss assumption can be accurate when the losses are extremely small or when the nodes implement ARQ schemes. Nevertheless, transmission losses can be easily accommodated in (4). In fact, the only modification is to scale q_k by $(1 - p_k^{loss})$, where p_k^{loss} stands for the packet loss probability.

The figure of merit to design the selective forwarder will be given by the accumulated importance of all messages *successfully* transmitted by the nodes. Accordingly, the total reward up to time k is defined as

$$t_k = \sum_{i=0}^{k} d_i r_i = \sum_{i=0}^{k} d_i q_i x_i u(e_i - c_{1,i}). \tag{5}$$

The selective forwarding policy is chosen in order to maximize the total expected reward, defined as

$$\mathbb{E}\{t_\infty\} = \mathbb{E}\left\{\lim_{k \to \infty} t_k\right\}. \tag{6}$$

Note that, since nodes have limited energy resources, the sum in (6) only contains a finite number of nonzero values (eventually, for some k, $e_k < \min_k c_{1,k}$, and $\forall k' \geq k$, we have $r_{k'} = 0$).

III. Optimal Selective Forwarding

A. Markov Decision Process

The tuple defined by $(\mathcal{S}, \mathcal{A}, P, r)$, where $\mathcal{S}$ is the set of states, $\mathcal{A} = \{0, 1\}$ is the set of possible decisions (actions), P is the transition probability measure given by (3) and r is the reward function, has the structure of a MDP. Moreover, since the action set $\mathcal{A}$ is finite, an optimal policy exists and it is Markovian. This means that there is an optimal policy such that, at any time k, the decision rule depends only on the state $\mathbf{s}_k$ [12]. Therefore, the sensor does not need to store the state history to make optimal decisions.

The following result, which provides an optimal selective forwarder, can be derived using some standard results from MDP models. Our proof is, however, self-contained. All expectations in the following are taken over $\mathbf{z}_k$, $c_{0,k}$, $c_{1,k}$ and q_k (which are the primary random variables in the model), unless otherwise stated through the conditional operators.

Theorem 1: *Let $\{\mathbf{z}_k, k \geq 0\}$ be a statistically independent sequence of importance values, and e_k the energy process given by (2). Consider the sequence of decision rules in the form*

$$d_k = u(Q_k(e_k, \mathbf{z}_k)x_k - \mu_k(e_k, \mathbf{z}_k)), \tag{7}$$

where $Q_k(e_k, \mathbf{z}_k) = \mathbb{E}\{q_k u(e_k - c_{1,k})|e_k, \mathbf{z}_k\}$ and threshold μ_k is defined recursively through the pair of equations

$$\mu_k(e, \mathbf{z}_k) = \mathbb{E}\{\lambda_{k+1}(e - c_{0,k}) - \lambda_{k+1}(e - c_{1,k})|\mathbf{z}_k\} \tag{8}$$

$$\begin{aligned}\lambda_k(e) =& (\mathbb{E}\{\lambda_{k+1}(e - c_{0,k})\} \\ &+ \mathbb{E}\{(Q(e, \mathbf{z}_k)x_k - \mu_k(e, \mathbf{z}_k))^+\})u(e),\end{aligned} \tag{9}$$

with $(z)^+ = zu(z)$, for any z.

Sequence $\{d_k\}$ is optimal in the sense of maximizing $\mathbb{E}\{t_\infty\}$ (with t_∞ given by (6)), among all sequences in the form $d_k = d_k(e_k, \mathbf{z}_k)$.

The auxiliary function $\lambda_k(e)$ represents the increment of the total importance that can be expected at time k, i.e.,

$$\lambda_k(e) = \sum_{i=k}^{\infty} \mathbb{E}\{d_i q_i x_i u(e_i - c_{1,i})|e_k = e\}. \tag{10}$$

Proof: See Appendix 1. ■

It is interesting to rewrite (7) as $d_k = u\left(Q_k(e_k, \mathbf{z}_k) - \mu_k(e_k, \mathbf{z}_k)/x_k\right)$, which expresses the node decision as a comparison of Q_k with a threshold inversely proportional to the importance value, x_k. This result is in agreement with our previous models in [16].

Note that (8) and (9) do not state a forward recursion (λ_{k+1} vs. λ_k) but a backward recursion (λ_k vs. λ_{k+1}). This makes the direct application of these equations impossible in a general non-stationary environment, because to compute λ_0 the importance distribution $\forall k$ should be known at time $k = 0$. The boundary conditions determining the solution to these recursive equations are given by $\lambda_k(e) = 0$, for any k and any $e < 0$, which are implicit in the factor $u(e)$ in (9).

Theorem 1 is general and holds for any energy cost and importance distribution, but it does not provide a clear intuition about the impact of the available energy and the distribution of x_k on the design of the optimal forwarding scheme. Due to this, in the reminder of the paper we will place attention on particular cases that will lead us to tractable closed-form solutions and useful insights.

B. Stationarity

If the statistical distributions of $\mathbf{z}_k$, $c_{0,k}$ and $c_{1,k}$ do not depend on k, then μ_k does not depend on k [c.f. (8) and (9)]. In this case, the following result can be shown:

Theorem 2: *Under the conditions of Th. 1, if the following conditions hold: (i) the statistical distribution of $\mathbf{z}_k$, $c_{0,k}$ and $c_{1,k}$ is independent of k; (ii) $P\{c_{i,k} > \epsilon\} = 1$, for $i = 0, 1$, some $\epsilon > 0$ and any $k \geq 0$; and (iii) $Q_k(e, \mathbf{z}) = Q(e, \mathbf{z})$ (i.e., Q_k does not depend on k), then the sequence of decision rules*

$$d_k = u(Q(e_k, \mathbf{z}_k)x_k - \mu(e_k, \mathbf{z}_k)), \tag{11}$$

where

$$\mu(e, \mathbf{z}_k) = \mathbb{E}\{\lambda(e - c_{0,k})|\mathbf{z}_k\} - \mathbb{E}\{\lambda(e - c_{1,k})|\mathbf{z}_k\} \tag{12}$$

$$\begin{aligned}\lambda(e) = &\left(\mathbb{E}\{\lambda(e - c_{0,k})\}\right. \\ &\left.+ \mathbb{E}\{(Q(e_k, \mathbf{z}_k)x_k - \mu(e, \mathbf{z}_k))^+\}\right)u(e),\end{aligned} \tag{13}$$

is optimal in the sense of maximizing $\mathbb{E}\{t_\infty\}$ (with t_∞ given by (6)), among all sequences in the form $d_k = d_k(e_k, \mathbf{z}_k)$.

Proof: See Appendix 2. ■

Strictly speaking, in most scenarios involving multiple sensors, the stationarity assumption is not true: The importance distribution of messages generated at source nodes could change along time, and even if it does not, the importance distribution of messages arriving to a given node may be time-variant, because it depends on the forwarding policies used by neighboring nodes, which depend on the available energy [c.f. either (8) or (12)], which reduces with time. However, the theoretical analysis and the experimental work in [1] suggests that, for medium to high values of available energy, the optimal forwarding scheme is usually not very sensitive to energy changes (which motivates the use of constant thresholds in the following section) and, thus, the stationarity assumption becomes a reasonable approximation during most of the time, with discrepancies arising only when the network is close to expire. From a computational point of view, the stationarity assumption is a good alternative to make the design of forwarding policies tractable.

C. Asymptotic analysis: constant threshold

Our experimental work in [1] shows that, for large values of the available energy, e, and for $Q(e, \mathbf{z}) = 1$ (i.e. for situations where the neighboring nodes always transmit the incoming messages), the optimal threshold converges to a constant value, and the expected reward tends to grow linearly. Both behaviors are closely related because, as (8) shows, the optimal threshold is the difference between two expected rewards. This situation can also be observed when $Q(e, \mathbf{z})$ does not depend on e. In that cases, we will write $Q(\mathbf{z}) = Q(e, \mathbf{z})$. In this section, we discuss the asymptotic behavior of any selective forwarder in the stationary case. To do so, we prove the following:

Theorem 3: *Assume that the conditions of Th. 2 hold and $\mathbb{E}\{x_k\} < \infty$. If $\lim_{e\to\infty} \mu(e, \mathbf{z})$ and $\lim_{e\to\infty} Q(e, \mathbf{z})$ exist, then*

$$\lim_{e\to\infty} \mu(e, \mathbf{z}) = (\mathbb{E}\{c_1|\mathbf{z}\} - \mathbb{E}\{c_0|\mathbf{z}\})\tau, \tag{14}$$

where τ is a solution of

$$\mathbb{E}\{c_0\}\tau = \mathbb{E}\{(xQ(\mathbf{z}) - (\mathbb{E}\{c_1|\mathbf{z}\} - \mathbb{E}\{c_0|\mathbf{z}\})\tau)^+\}. \quad (15)$$

Moreover, if $\mathbb{E}\{c_1|\mathbf{z}\} \geq \mathbb{E}\{c_0|\mathbf{z}\}$, for any $\mathbf{z}$, this solution is unique.

Proof: See Appendix 3. ■

An important consequence of Theorem 3 is that, if $\lim_{e\to\infty} \mu(e, \mathbf{z})$ exists, it must be equal to (14). The main idea behind the selective forwarding algorithms explored in this paper is to replace the optimal rules in (11) - (13) by their asymptotic approximations based on (14) and (15). Our experimental work in [1] suggests that this is a good choice provided that there is enough energy for a reasonable number of transmissions. If the node has some battery for only a few transmissions, the forwarding threshold should start to oscillate and decreases in some way defined by (11) and (12), but the computational cost of computing such threshold is high. The design of computationally efficient forwarding policies for small batteries is an open issue.

Thus, (14) is the basis of the adaptive procedure proposed in the next section.

IV. Parameter Estimates

A. Estimating asymptotic thresholds

The optimal threshold depends on the distribution of message importances, which in practice may be unknown. To bypass this problem, we can try to estimate parameter τ in (15) and replace the optimal threshold function by its asymptotic limit. Parameter τ can be estimated in real time based on the available information at time k: $\{(\mathbf{z}_\ell, q_\ell), \ell = 0, \ldots, k\}$.

Defining the mean energy difference

$$\Delta(\mathbf{z}) = \mathbb{E}\{c_1|\mathbf{z}\} - \mathbb{E}\{c_0|\mathbf{z}\}, \quad (16)$$

we can estimate the expected value on the right-hand side of (15) as $\mathbb{E}\{(xQ(\mathbf{z}) - \Delta(\mathbf{z})\tau)^+\} \approx m_k$ where

$$\begin{aligned} m_k =& \frac{1}{k}\sum_{i=1}^{k}(x_i Q(\mathbf{z}_i) - \Delta(\mathbf{z}_i)\tau)^+ \\ =& \left(1 - \frac{1}{k}\right) m_{k-1} + \frac{1}{k}(x_k Q(\mathbf{z}_k) - \Delta(\mathbf{z}_k)\tau)^+. \end{aligned} \quad (17)$$

(15) can be used to estimate τ at time k as $\tau_k = m_k/\epsilon_0$, where $\epsilon_0 = \mathbb{E}\{c_0\}$. Using (17), we get

$$\tau_k = (1 - 1/k)\,\tau_{k-1} + (x_k Q(\mathbf{z}_k) - \Delta(\mathbf{z}_k)\tau_{k-1})^+/k\epsilon_0, \quad (18)$$

where we have replaced τ (which is unknown) in the right-hand side of (17) with τ_{k-1}.

If the mean energy difference, $\Delta(\mathbf{z})$ is unknown, it can be estimated from data. Making the simplifying assumption that the energy cost does not depend on $\mathbf{z}$ (which may be realistic, for instance, if $\mathbf{z}$ only contains the importance value), it can be estimated as the difference of the average costs of past decisions. For instance $\mathbb{E}\{c_1\} \approx \left(\sum_{i=1}^{k} d_i c_1^i\right) / \left(\sum_{i=1}^{k} d_i\right)$.

B. Estimating the success index

A simple estimate of the selective communication policy $Q_k(e_k, \mathbf{z}_k) = \mathbb{E}\{q_k u(e_k - c_{1,k})|e_k, \mathbf{z}_k\}$ can be derived by assuming that: a) it does not depend on e_k (i.e., the subsequent forward/discard decision taken at the receiving node is independent of the energy state at the transmitting node), and b) each node can know about the success of the transmission. When q_k represents the local success index, each node is able to listen to the retransmission of a message that has been previously sent (i.e. each node can observe q_k when $d_k = 1$). When q_k represents the global success index, the sink node has to acknowledge the reception of messages back through the routing path, so that nodes can observe q_k and use it for their estimation algorithm. Following an approach similar to that in [16] and [17], Q_k can be estimated by means of the parametric model

$$Q_k(\mathbf{z}_k, w, b) = P\{q_k = 1|x_k, w, b\} = (1 + \exp[-w(x_k - b)])^{-1}. \quad (19)$$

Note that the only component of $\mathbf{z}_k$ which has some influence on Q_k in (19) is x_k. For positive values of w, Q_k increases monotonically with x_k, as expected from the node behavior. We estimate parameters w and b via ML (maximum likelihood) using the observed sequence of neighbor decisions $\{q_k\}$ and importance values $\{x_k\}$, by means of stochastic gradient learning rules

$$w_{k+1} = w_k + \eta(q_k - Q_k(x_k, w_k, b_k))(x_k - b_k) \quad (20)$$

$$b_{k+1} = b_k - \eta(q_k - Q_k(x_k, w_k, b_k))w_k \quad (21)$$

where learning step η is a free parameter of the rules. Clearly, the selection of η entails a trade-off between the speed of convergence (higher for large η) and the stability of the adaptive rule (better for small η).

V. Numerical Experiments and Results

In this section we analyze the performance of selective forwarders in different scenarios through simulations based on Matlab. First, we describe some common features of the experimental setup.

1) We have used a simple deterministic energy model given by three constant and known parameters: (a) E_I, energy spent on idle states; (b) E_R, energy spent on receiving or sensing a single message; and (c) E_T, energy spent on transmitting a single message. According to this, for $x_k > 0$, energy consumption in (2) is constant and deterministic, so that $c_{1,k} = E_T + E_R$ and $c_{0,k} = E_R$. On the other hand, if $x_k = 0$, $c_{1,k} = c_{0,k} = E_I$. Note that for simplicity, we assumed that the value of E_R is the same no matter if data have been taken from the sensing device or received from other node. Energy values are set to $E_T = 4$, $E_R = 1$ and $E_I = 0$.
2) Nodes are homogeneous and their initial level of battery is the same except for the sink, which has unlimited power supply. Nodes keep working until their batteries expire. The network dies when all the sink neighbors have died.
3) Sources are selected at random. Nodes are assumed to have identical transmission radii. They can communicate

only if they are within mutual transmission range. Under this assumption and by listening to the channel, nodes are able to update their information estimates. Moreover, each node knows the location of its neighbors, the sink and itself.

4) Though the selective forwarding strategies can be implemented in any routing algorithm, we have used the greedy forwarding scheme [18] for simplicity. This scheme selects the neighbor geographically closest to the sink as the next hop of the message. With the same aim, link losses have not been included in the model.
5) Up to our knowledge, there are no other proposals in the literature oriented to maximize the important sum or any related measure. The only exception may be the work in [11], which uses the same paradigm (MDP), but it is oriented to a completely different scenario (replenishable sensors) and cannot be expected to have a good behavior in sensors with finite lifetime. On the other hand, making a comparison with an oracle, although interesting, it is likely impossible in practice because of the computational complexity. Note that the oracle should take into account all possible decision chains, which grow exponentially with time and network size. Therefore, to evaluate the behavior of the developed schemes, different types of selective nodes are implemented. *Non-Selective sensor* (NS). The sensor does not censor any message. *Adaptive selective Transmitter* (AT), which uses the zero-order success index. *Local selective Forwarder* (LF). It computes the forwarding threshold according to (8) and (9), considering the local success index. *Global selective Forwarder* (GF), which uses the global success index.
The only information used by nodes to make decisions is the importance value. In other words, $\mathbf{z}_k = x_k$. Moreover, selective sensors compute the forwarding threshold adaptively via (18).
6) Performance is assessed in terms of the importance sum of all messages received by the sink, the mean value of the received importance, the number of receptions at the sink, and the number of *generated* messages (the latter amounts to measure the network lifetime).
7) Experimental results are averaged over 50 different topologies with different traffic patterns.

A. Nodes with full information

1) Chain network: With the aim of illustrating the merits of the selective forwarding policies, we have selected a first simplified setup where 30 nodes (numbered from 1 to 30 from left to right, being node 30 the sink) are equidistantly placed in a row. Each node can only communicate with its two adjoint neighbors. This simple configuration tries to emulate the scenario where nodes placed closer to the sink have more activity than those far-off located, representing a bottleneck in the routing path. In this setting we assume that selective forwarding nodes (LF and GF) know the forwarding threshold used by their neighbors. Node batteries are initially charged with 3000 units. All nodes generate messages whose importance values follow a uniform distribution $U(0, 1)$, except for node 29 (connected to the sink), which generates importance values according to a $U(1, 2)$ distribution.

TABLE I
AVERAGED PERFORMANCE RESULTS IN A CHAIN NETWORK OF 30 EQUALLY-SPACED NODES. IMPORTANCE VALUES ARE GENERATED ACCORDING TO A UNIFORM DISTRIBUTION.

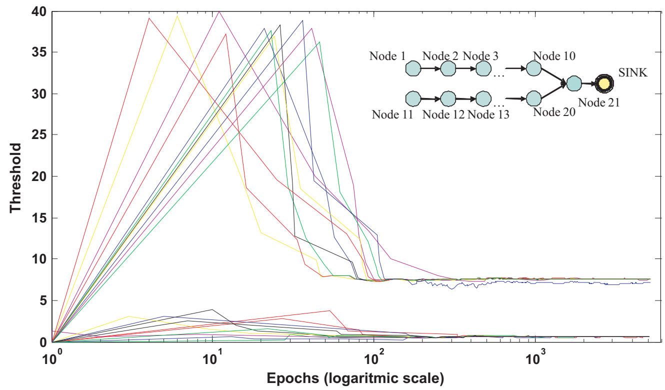
	Total Import. Received	Importance mean value	Number of Receptions	Number of gen. Messages
Type NS	321.04	0.54	599.00	600.00
Type AT	552.44	0.96	577.42	2623.60
Type LF	910.66	1.60	567.58	21170.40
Type GF	910.66	1.60	567.58	21170.40

This scenario has been designed ad-hoc to make clear the differences among the diverse types of sensors tested. The numerical results are listed in Table I and reveal that:

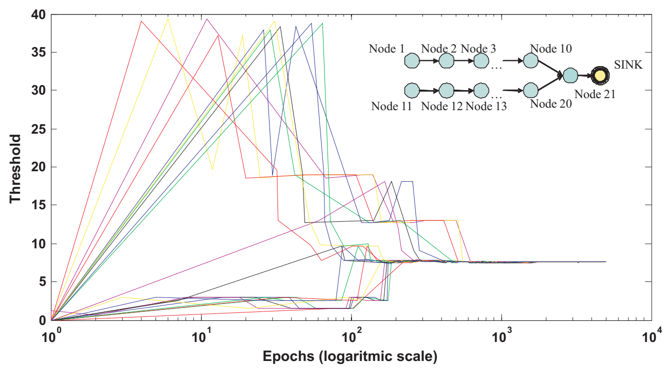
- All selective communication strategies (AT, LF and GF) outperform non-selective forwarding (NS). Despite the fact that the latter delivers more messages to the sink, the total importance sum is lower and so the network lifetime (NS nodes waste energy sending low importance messages).
- LF and GF nodes get around 65% higher importance sum than that achieved by AT nodes. Also, the higher mean value of the messages received by the sink implies that selective forwarders are much more selective, enlarging the network lifetime. This is not surprising: AT nodes do not pay attention to the final destination of messages, and keep sending messages that will not be delivered (node 29 tends to reject most messages from other nodes). On the contrary, GF nodes are aware of its higher selectivity, and inhibit transmissions that they know will not succeed.
- Interestingly, LF and GF results are identical. GF nodes have direct access to the threshold set by node 29, and can inhibit transmissions. LF nodes do not have this information directly, but it is propagated backwards: nodes notice that node 29 is not forwarding messages below a certain threshold, and set their own threshold accordingly. Thus, despite the fact that LF nodes only use local information, it has the global effect of removing all nonimportant traffic from the network.

2) Threshold evolution in a double branch network: The time evolution of the forwarding threshold in a two-branch network is also illustrative of the behavior of selective sensors. The network sketch is represented at the top-right corner of both plots in Fig. 1. Nodes 1-10 and 11-20 form two branches of nodes placed equidistantly in a line. Nodes 10 and 20 are connected to node 21, which delivers all network messages to the sink. While nodes 1-10 generate low importance messages ($U(0, 1)$), nodes 11-20 generate messages of high importance ($U(9, 10)$). Node 21 generates a mixture of both distributions. This scenario represents the arrival of two flows of different prioritized messages at a common node in the routing path to the sink.

Figures 1(a) and 1(b) plot the threshold evolution for AT and LF nodes, respectively. Fig. 1(a) shows that the lines of nodes 1-10 converge to a low threshold, while those of nodes



(a)



(b)

Fig. 1. The decision threshold evolution for Adaptive Transmitters (a) and Local Forwarders (b) as a function of the time in a simulation run, represented in a X-axis logarithmic scale. Uniform importance distributions are assumed.

11-20 converge to a high threshold. The threshold of node 21 converges to a high value, which means that all messages arriving to node 21 from the low importance branch will not be delivered to the sink.

In contrast, all nodes in Fig. 1(b) follow a similar trend. Thresholds tend to converge to the value established by node 21, as a consequence of the same backward propagation mechanism observed in the chain network. The low prioritized traffic is again removed from the network.

3) Threshold evolution after battery depletion: Another simple setup will serve to show how selective forwarders adapt their thresholds to network topology changes. Fig. 2 shows the threshold evolution of a sensor network (sketched at the top-right) with 3 nodes and the sink. With the aim of analyzing the adaptive behavior of thresholds, nodes 1 and 3 are charged with more batteries than node 2. Nodes 1, 2 and 3 generate messages according to exponential importance distribution with means $m_1 = 1.59$, $m_2 = 2.57$ and $m_3 = 0.95$, respectively (note that $m_3 < m_1 < m_2$, the specific values of the means not being relevant).

Although node 1 can transmit messages to nodes 2 and 3 (which are linked to the sink), it will do it initially to node 2 (according to the greedy forwarding routing policy). Since nodes are LF, the thresholds of nodes 1 and 2 get rapidly coupled (i.e., they converge to the same value); Fig. 2 illustrates this fact. Once node 2 runs out of battery, node 1 starts routing messages to the sink via node 3, which still has energy. From that point on, node 1 reduces its threshold,

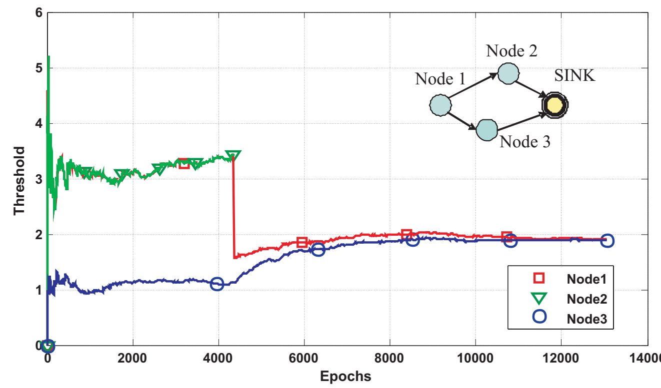


Fig. 2. The decision threshold for Local Forwarders as a function of time in a network topology composed of 4 nodes. Different exponential importance distributions are assumed.

because node 3 is less selective, and node 3 adapts its threshold according to the new messages arriving from node 1, which have higher importance values (because $m_3 < m_1$). As a result, nodes 1 and 3 get coupled.

B. Nodes with incomplete information

In more realistic settings, nodes must acquire the information about neighboring thresholds, either by estimating (learning) it from side information or by paying some cost (energy, bandwidth) to learn it. The aim of this subsection is to analyze the performance of sensors with incomplete information and compare it with results of sensors with full information (-FI).

The setup is the same linear arrangement of 30 nodes described in Section V-A1, but with message importances drawn from exponential distributions. The mean of each distribution is taken from the increasingly sorted samples generated randomly from another exponential distribution with mean 2. Nodes manage to get threshold information from their neighbors following two different approaches:

- *Based on learning*. Nodes estimate the parameters of the probability model Q_k in (19) using learning rules, (20) and (21) (-EST). The success index, q_k, of previous transmissions, which is required to apply these rules, can be obtained in different ways: we assume that LF nodes can listen to the channel to check if a neighboring node forwards a message or not, and also, that GF nodes receive acknowledgments from the sink whenever a message arrives successfully.
- *Based on feedback*. Nodes transmit threshold values to their neighbors. Since transmitting the threshold every time it changes is energy expensive, we have explored two ways to reduce communication overheads: (i) to send the threshold to all their neighbors every time the current value differs more than a certain fixed amount (denoted by β_μ) from the last transmitted value (-VAR), or (ii) to broadcast the threshold to all neighbors periodically using beacons (-BEAC).

In the experiments, the value of β_μ is set to 3.5, the beacon interval to 2500 epochs, and the value of η in (20) and (21) to 0.04 for LF nodes and 0.009 for GF nodes. These values

TABLE II
AVERAGED PERFORMANCE RESULTS IN A CHAIN NETWORK OF 30 EQUALLY-SPACED NODES. IMPORTANCES FOLLOW EXPONENTIAL DISTRIBUTIONS.

	Total Import. Received	Importance mean value	Number of Receptions	Number of gen. Messages
Type NS	1074.02	1.79	599.00	600.00
Type AT	6962.02	14.55	480.44	15638.10
LF-FI	7698.31	15.73	491.54	20563.78
LF-EST	7356.90	15.76	468.56	19911.62
LF-VAR	7696.91	16.80	459.32	24661.00
LF-BEAC	7429.23	15.41	484.10	19243.56
GF-FI	7699.94	15.73	491.40	20597.22
GF-EST	7319.05	15.13	485.82	17415.22

have been adjusted off-line to gauge the potential advantages of schemes relying on incomplete information. The automatic assignment of these parameters, which is certainly important from a practical implementation perspective, goes beyond the scope of this paper.

Results are summarized in Table II. As expected, the best performance is achieved by LF-FI networks. The approaches that rely on feedback also provide good performance, always exceeding the results of the AT: an improvement around 5.7% for the estimate-based approach, and around 6.7% for the use of beacons. Increasing the frequency of beacons causes a significant decrease of the importance sum due to the fast energy expense. Reporting significant threshold variations (-VAR) provides an improvement around 10.5%, performing similarly to LF-FI sensor networks. Decreasing the value of β_μ causes very frequent threshold update reports due to the initial instability in thresholds, reducing quickly the batteries. In all cases, the lifetime of LF sensor networks is higher than that of AT sensor networks, as the higher number of generated messages shows. Even more, in some cases the number of messages received by the sink is slightly higher than for AT networks. The same conclusions can be extrapolated to GF networks. LF-FI and GF-FI results are practically identical. GL-EST results are slightly worse than those achieved by the LF nodes, but very close to LF-EST.

C. Networks with arbitrary topologies

To analyze the behavior of selective forwarders in a more realistic scenario, we simulate a network of 140 nodes scattered in a square field with corners (0,0) and (L,L), with $L = 150$. Nodes are denser deployed near the sink, tailing off towards the edges. Nodes report the information to a unique sink, located at (L,$L/2$). Fig. 3 illustrates a sketch of the sensor network deployment. Each node generates importances following an exponential distribution with different mean, whose values were randomly generated from another exponential distribution with mean 2. Messages are equally generated in the three regions and node batteries are charged to 1500 units. Results are averaged over 20 different topologies, where the average depth of the network (number of hops required to reach the edges from the sink) is 7. Note that if the depth of the network increases substantially, threshold information in LF nodes will propagate slower towards the edges of the sensor field and could eventually cause a degradation in the results, especially when batteries are close to run out, compared with GF nodes.

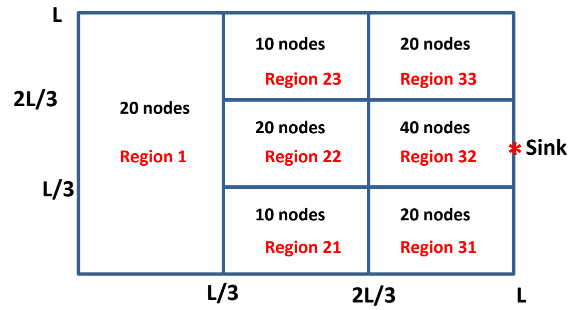


Fig. 3. Sensor Network deployment sketch.

TABLE III
AVERAGED PERFORMANCE RESULTS IN AN ARBITRARY NETWORK TOPOLOGY OF 140 NODES. IMPORTANCES ARE GENERATED ACCORDING TO EXPONENTIAL DISTRIBUTIONS OF DIFFERENT MEAN.

	Total Import. Received	Importance mean value	Number of Receptions	Number of gen. Messages
NS	10328.62	1.92	5414.00	9292.20
AT	39105.03	10.13	3934.35	66630.15
LF-FI	45786.56	11.04	4223.95	101537.35
LF-EST	40578.81	10.36	3992.50	72406.10
LF-VAR	40075.74	11.32	3619.00	102801.30
LF-BEAC	40955.17	11.05	3766.90	86856.55
GF-EST	40957.99	10.42	4007.50	72997.05

Table III compares the performance for all types of nodes. Parameter η is fixed to 0.008 for LF and GF nodes. The value of β_μ is set to 3.5, and the beacon interval to 13000 epochs. The conclusions are similar to those of section V-B. The importance sum of messages arriving to the sink in AT networks is around 280% higher than that of NS networks. The improvement is even higher when deploying selective forwarders (around 293% for LF estimate-based nodes and 343% for LF-FI nodes). Also, note that the network lifetime is at least 6 times longer than that of NS networks.

The comparative analysis of selective sensors shows again that LF nodes outperform AT nodes. Not only are the most relevant messages prioritized to arrive earlier to the sink (shown through the importance mean value of messages received by the sink), but also the sensor network lifetime is enlarged, beating the average value obtained by selective AT networks. Specifically, the best performance corresponds to LF-FI sensor networks. The importance sum is 17% better than that of AT sensors. As nodes initially lack of information from their neighbors, the approximate approaches also yield a reasonable performance. Among the proposed techniques, we would like to emphasize the good performance achieved by the estimate-based proposal, both with local and global optimization. Even, GF-EST results are slightly better than those of LF with incomplete information.

We have also analyzed the threshold values in different regions of the sensor field for the same setup. Averaged results for AT and LF-FI sensor nodes are shown in Table IV. In

TABLE IV
AVERAGED THRESHOLD VALUE IN DIFFERENT REGIONS OF AN ARBITRARY NETWORK TOPOLOGY OF 140 NODES. IMPORTANCES ARE GENERATED ACCORDING TO EXPONENTIAL DISTRIBUTIONS OF DIFFERENT MEAN.

	R1	R21	R22	R23	R31	R32	R33
AT	2.79	3.36	3.85	3.63	3.05	4.88	3.39
LF-FI	11.20	9.34	10.48	10.85	7.42	7.89	8.56

TABLE V
AVERAGED REMAINING BATTERY IN DIFFERENT REGIONS OF AN ARBITRARY NETWORK TOPOLOGY OF 140 NODES. IMPORTANCES ARE GENERATED ACCORDING TO EXPONENTIAL DISTRIBUTIONS OF DIFFERENT MEAN.

	R1	R21	R22	R23	R31	R32	R33
AT	0	7.07	6.65	6.45	237.20	113.16	244.84
LF-FI	0	221.13	199.42	222.40	641.16	289.81	659.52

general, AT nodes belonging to regions closer to the sink set higher thresholds than those faraway located. As AT nodes set threshold values independently, the furthest nodes (which only have to transmit their own generated traffic) set the lowest thresholds. Insofar as nodes approach to the sink, threshold values are slightly increased as a consequence of receiving messages with clipped importances from the furthest nodes. On the contrary, the opposite effect is observed for LF-FI nodes, i.e., threshold values are lower in those regions placed near the sink. It would not make sense that nodes approaching to the sink set high thresholds because faraway generated messages will be never forwarded. Naturally, the most selective nodes in the field are those located near the edges because they ensure the forwarding of their own generated traffic and thus, they avoid wasting energy transmitting messages that will not be forwarded. Recall that nodes generate messages of different importances and moreover, set their thresholds also taking into account neighbor behavior (whose information is backward propagated).

Finally, Table V lists the sensors remaining battery (averaged) for each region. Results are shown for AT and LF-FI nodes. Clearly, AT nodes do their best concerning energy consumption but LF-FI nodes are those that make a better use of energy resources.

D. Stochastic energy costs in networks with arbitrary topology

Up to this point, a simple deterministic energy model with constant energy costs was assumed. But $c_{1,k}$ and $c_{0,k}$ can be stochastic processes. This fact allows integrating in the sensor model the idea of nodes consuming a different amount of energy at every state. Energy consumption may depend on factors such as the amount of time spent in each state (which in fact is closely linked to messages of different lengths as a consequence (or not) of having different priorities) or the inter-sensor distances (transmitting a message to faraway nodes implies a higher consumption), taken into account in the energy model presented in [19]. However, we consider a more general consumption model, exposed in Section II, which includes stochastic energy costs. For that purpose, energy estimates (E_T, E_R and E_I) are samples of noncentral Chi-square distributions of one degree of freedom. The other free parameter of the distribution, λ, takes the value 3.5 for E_T, 1 for E_R and 2 for E_I. To compute the forwarding threshold, the average value of the energy costs is needed. To save memory resources, all estimates are based on an Exponential Weighted Moving Average (EWMA). The node probability of being in idle state, P_I, is computed based on the number of idle states and the total data events.

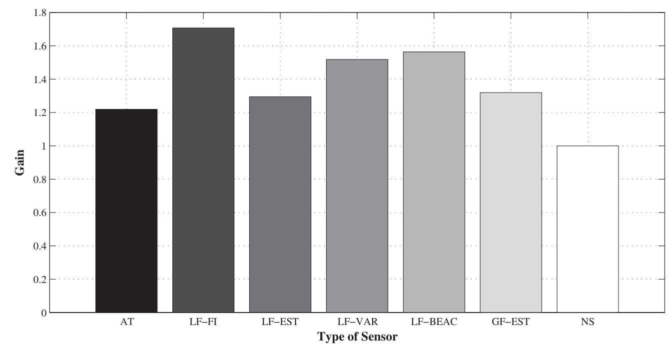


Fig. 4. Gain of the selective forwarding policies under exponential importance distributions when energy costs are stochastic.

In the experiments, the network is composed of 150 nodes provided with $E = 10,000$ units and they are randomly scattered in a square field. Parameter η is fixed to 0.1 for LF and GF nodes. The value of β_μ is set to 1.5 and the beacon interval to $2,000$ epochs. Results are averaged over 50 different topologies and the depth of the network is, on average, 8 hops.

The gain for different types of sensors is depicted in Fig. 4. It highlights the advantage of including selective nodes in sensor networks. The best performance corresponds to the LF-FI node. Nevertheless, remark the good behavior achieved by LF-VAR and LF-BEAC nodes, which approach to the results got by the LF-FI node. GF-EST gets a slightly better results than LF-EST, what makes good sense to the fact that the global forwarder performs rather better than the other two types of selective nodes.

VI. CONCLUSIONS

This paper has introduced several selective message forwarding policies to save energy and extend the lifetime of WSN. Messages, which were assumed to be graded with an importance value and which could be eventually discarded, were transmitted by sensor nodes according to a forwarding policy, which considered consumption patterns, available energy resources in nodes, the importance of the current message and the statistical description of such importances.

Forwarding schemes were designed for three different scenarios: 1) when sensors maximize the importance of their own transmitted messages (no information from other nodes is available); 2) when sensors maximize the importance of messages that have been successfully retransmitted by at least one of its neighbors (nodes need to know/estimate if the message was retransmitted); and 3) when sensors maximize the importance of the messages that successfully arrive to the sink (nodes need to know if the message arrived to the sink). Interestingly, the structure of the optimal scheme was the same in all three cases and consisted of comparing the received importance to a forwarding threshold. The expression to find the optimum of the threshold varies with time and is slightly different for each scenario. The developed schemes were optimal from an importance perspective, efficiently exploited the energy resources, entailed very low computational complexity and were amenable to distributed implementation, all desirable characteristics in WSN. The three schemes have been compared under different criteria. From an overall network efficiency perspective, the first scheme performed worse that its counterparts, but it required less signaling overhead. On the contrary, the last scheme was the best in terms of network performance, but it required the implementation of feedback messages from the sink to the nodes of the WSN. Numerical results showed that for the tested cases the differences among the three schemes were small -with schemes two and three performing evenly. From a practical perspective, this suggests that the second scheme, which is just slightly more complex than the first one, can be the best candidate in most practical networks (especially in new deployments). Similarly, from a modeling point of view, the results indicate that when nodes have access to non-local information, information of first order neighbors may be enough.

Appendix A: Proof of Theorem 1

The cumulative importance at time k, given by (5), can be expressed recursively as $t_k = t_{k-1} + d_k r_k$ and, for any $k > 0$, the accumulated importance can be expressed as

$$t_\infty = \sum_{i=0}^{\infty} d_i r_i = t_{k-1} + \sum_{i=k}^{\infty} d_i r_i. \tag{22}$$

Since, for any k, $\mathbb{E}\{t_\infty\} = \int \mathbb{E}\{t_\infty | e_k, \mathbf{z}_k\} dP(e_k, \mathbf{z}_k)$, maximizing $\mathbb{E}\{t_\infty\}$ is equivalent to maximize, for each k, $\mathbb{E}\{t_\infty | e_k, \mathbf{z}_k\}$, which can be expressed as

$$\begin{aligned}\mathbb{E}\{t_\infty | e_k, \mathbf{z}_k\} =& \mathbb{E}\{t_{k-1} | e_k, \mathbf{z}_k\} + d_k \mathbb{E}\{r_k | e_k, \mathbf{z}_k\} \\ &+ \sum_{i=k+1}^{\infty} \mathbb{E}\{d_i r_i | e_k, \mathbf{z}_k\},\end{aligned} \tag{23}$$

where we have used the fact that d_k is a deterministic function of e_k and $\mathbf{z}_k$. Also, it is useful to write, for any $i > k$,

$$\begin{aligned}\mathbb{E}\{d_i r_i | e_k, \mathbf{z}_k\} =& (1 - d_k) \mathbb{E}\left\{d_i r_i | e_k, \mathbf{z}_k, d_k = 0\right\} \\ &+ d_k \mathbb{E}\left\{d_i r_i | e_k, \mathbf{z}_k, d_k = 1\right\}.\end{aligned} \tag{24}$$

Taking into account that

$$\begin{aligned}&\mathbb{E}\{d_i r_i | e_k = e, \mathbf{z}_k, d_k = 0\} \\ &\quad= \int \mathbb{E}\{d_i r_i | e_k = e, \mathbf{z}_k, d_k = 0, c_{0,k}\} \\ &\qquad \times dP(c_{0,k} | e_k = e, \mathbf{z}_k, d_k = 0) \\ &\quad= \int \mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}\} dP(c_{0,k} | \mathbf{z}_k) \\ &\quad= \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}\} | \mathbf{z}_k\}\end{aligned} \tag{25}$$

where we have used the fact that the data sequence, $\mathbf{z}_k$, is statistically independent, so that both d_i and r_i are independent of $\mathbf{z}_k$, for $i > k$, and we can remove $\mathbf{z}_k$ in the inner conditional expectations in (25). The outer expectation should be taken over $c_{0,k}$. Proceeding in an analog manner with $\mathbb{E}\{d_i r_i | e_k = e, \mathbf{z}_k, d_k = 1\}$, we arrive at

$$\begin{aligned}&\mathbb{E}\{d_i r_i | e_k = e, \mathbf{z}_k\} \\ &\quad= (1 - d_k) \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}\} | \mathbf{z}_k\} \\ &\qquad + d_k \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{1,k}\} | \mathbf{z}_k\}.\end{aligned} \tag{26}$$

Upon defining $Q_k(e_k, \mathbf{z}_k) := \mathbb{E}\{q_k u(e_k - c_{1,k}) | e_k, \mathbf{z}_k\}$ and replacing (26) into (23), we get

$$\begin{aligned}\mathbb{E}\{t_\infty | e_k = e, \mathbf{z}_k\} =& \mathbb{E}\{t_{k-1} | e_k = e, \mathbf{z}_k\} \\ &+ (1 - d_k) \sum_{i=k+1}^{\infty} \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}\} | \mathbf{z}_k\} \\ &+ d_k x_k Q_k(e, \mathbf{z}_k) \\ &+ d_k \sum_{i=k+1}^{\infty} \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{1,k}\} | \mathbf{z}_k\}.\end{aligned} \tag{27}$$

Using the definition of λ_{k+1} in (10), the expected accumulated importance can be written as

$$\begin{aligned}\mathbb{E}\{t_\infty | e_k = e, \mathbf{z}_k\} =& \mathbb{E}\{t_{k-1} | e_k = e, \mathbf{z}_k\} \\ &+ (1 - d_k) \mathbb{E}\{\lambda_{k+1}(e - c_{0,k}) | \mathbf{z}_k\} \\ &+ d_k x_k Q_k(e, \mathbf{z}_k) + d_k \mathbb{E}\{\lambda_{k+1}(e - c_{1,k}) | \mathbf{z}_k\}.\end{aligned} \tag{28}$$

Defining $\mu_k(e_k, \mathbf{z}_k)$ as in (8), we get

$$\begin{aligned}\mathbb{E}\{t_\infty | e_k = e, \mathbf{z}_k\} =& \mathbb{E}\{t_{k-1} | e_k = e, \mathbf{z}_k\} + \mathbb{E}\{\lambda_{k+1}(e - c_{0,k}) | \mathbf{z}_k\} \\ &+ d_k (x_k Q_k(e, \mathbf{z}_k) - \mu_k(e, \mathbf{z}_k))).\end{aligned} \tag{29}$$

Clearly, the decision rule given by $d_k = 1$ as soon as $x_k Q_k(e_k, \mathbf{z}_k) \geq \mu_k(e_k, \mathbf{z}_k)$ (so as to maximize the third term in (29)) and $d_k = 0$ otherwise, is optimal in the sense of maximizing $\mathbb{E}\{t_\infty | e_k, \mathbf{z}_k\}$. Therefore, $d_k = u(x_k Q_k(e_k, \mathbf{z}_k) - \mu_k(e_k, \mathbf{z}_k))$, where $u(\cdot)$ is the step function.

The recursive computation of $\lambda_k(e)$ in (9) is the only result that remains to be proved. To do so, we note that, for any $i > k$,

$$\begin{aligned}\mathbb{E}\{d_i r_i | e_k = e\} =& P_{0,k}(e) \mathbb{E}\{\mathbb{E}\{d_i r_i | e_k = e, d_k = 0\}\} \\ &+ P_{1,k}(e) \mathbb{E}\{\mathbb{E}\{d_i r_i | e_k = e, d_k = 1\}\} \\ =& P_{0,k}(e) \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}\} | e_k\} \\ &+ P_{1,k}(e) \mathbb{E}\{\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{1,k}\} | e_k\}\end{aligned} \tag{30}$$

where $P_{0,k}(e) = \Pr\{d_k = 0 | e_k = e\}$ and $P_{1,k}(e) = 1 - P_{0,k}(e)$. The outer expectations must be taken over $c_{0,k}$ and $c_{1,k}$. Using the definition of $\lambda_k(e)$ in (10) and capitalizing on (30), we find

$$\begin{aligned}\lambda_k(e) =& \sum_{i=k}^{\infty} \mathbb{E}\{d_i r_i | e_k = e\} = \mathbb{E}\{d_k r_k | e_k = e\} \\ &+ \sum_{i=k+1}^{\infty} \mathbb{E}\{\mathbb{E}\{d_i r_i | e_k = e, \mathbf{z}_k\}\}\end{aligned} \tag{31}$$

where the outer expectation applies over $\mathbf{z}_k$. Taking into account that d_k only depends on e_k and $\mathbf{z}_k$, the conditions in the inner expectation operators determine uniquely d_k and, thus, we can write

$$\begin{aligned}\lambda_k(e) =&\mathbb{E}\{d_k r_k | e_k = e\} \\ &+ \sum_{i=k+1}^{\infty} \mathbb{E}\Big\{(1-d_k)\mathbb{E}\{d_i r_i | e_{k+1} = e - c_{0,k}, \mathbf{z}_k\} \\ &+ d_k \mathbb{E}\{d_i r_i | e_{k+1} = e - c_{1,k}, \mathbf{z}_k\} | e_k = e\Big\} \\ =&\mathbb{E}\{d_k r_k | e_k = e\} + \mathbb{E}\{(1-d_k)\lambda_{k+1}(e-c_{0,k}) | e_k = e\} \\ &+ \mathbb{E}\{d_k \lambda_{k+1}(e - c_{1,k}) | e_k = e\} \\ =&\mathbb{E}\{\lambda_{k+1}(e-c_{0,k})\} + \mathbb{E}\{d_k(r_k - \mu_k(e_k, \mathbf{z}_k)) | e_k = e\} \\ =&\mathbb{E}\{\lambda_{k+1}(e-c_{0,k})\} + \mathbb{E}\{(x_k q_k u(e - c_{1,k}) - \mu_k(e, \mathbf{z}_k)) \\ &\cdot u(x_k Q_k(e, \mathbf{z}_k) - \mu_k(e, \mathbf{z}_k)) | e_k = e\} \\ =&\mathbb{E}\{\lambda_{k+1}(e-c_{0,k})\} + \mathbb{E}\{(x_k Q_k(e, \mathbf{z}_k) - \mu_k(e, \mathbf{z}_k)) \\ &\cdot u(x_k Q_k(e, \mathbf{z}_k) - \mu_k(e, \mathbf{z}_k))\}. \end{aligned} \tag{32}$$

Using (10), and taking into account that, if there is no available energy, no transmissions are possible, we can write $\lambda_k(e) = 0$, for any k and $e \leq 0$. Combining the latter with (32), we get (9), completing the proof.

Appendix B: Proof of Theorem 2

Take $\epsilon > 0$ such that, with probability 1, $c_{1,i} > \epsilon$ for any i. We prove the theorem by induction, by showing that $\lambda_k(e)$ does not depend on k for $e \leq n\epsilon$, for any n. This is true for $n = 0$, because $\lambda_k(0) = 0$. Now, let us assume that $\lambda_k(e)$ does not depend on k for $e \leq n\epsilon$. If $n\epsilon < e \leq (n+1)\epsilon$, by (8) we find that $\mu_k(e, \mathbf{z})$ does not depend on k. Thus, using (9), and taking into account that expectations are taken over $\mathbf{z}_k$, whose distribution does not depend on k, we find that $\lambda_k(e)$ does not depend on k, which completes the proof.

Appendix C: Proof of Theorem 3

Let define $h(e) = \mathbb{E}\{(Q(e, \mathbf{z}_k)x_k - \mu(e, \mathbf{z}_k))^+\}$ and note that $\mathbb{E}\{\lambda(e - c_0)\} = \int \lambda(e - c) p_{c_0}(c) dc = \lambda(e) * p_{c_0}(e)$, where '$*$' denotes convolution and $p_{c_0}(e)$ is the probability density function of c_0. Then, (13) can be written as $(\delta(e) - p_{c_0}(e)) * \lambda(e) = h(e)u(e)$, where subindex k in e_k and $c_{0,k}$ has been omitted for simplicity. Similarly, (12) can be written as $\mu(e, \mathbf{z}) = \lambda(e) * (p_{c_0|\mathbf{z}}(e) - p_{c_1|\mathbf{z}}(e))$. Hence, $(\delta(e) - p_{c_0}(e)) * \mu(e, \mathbf{z}) = (h(e)u(e)) * (p_{c_0|\mathbf{z}}(e) - p_{c_1|\mathbf{z}}(e))$. Convolving the latter with $u(e)$ yields

$$(u(e) - P_{c_0}(e)) * \mu(e, \mathbf{z}) = (h(e)u(e)) * (P_{c_0|\mathbf{z}}(e) - P_{c_1|\mathbf{z}}(e)), \tag{33}$$

where P_{c_0}, $P_{c_0|\mathbf{z}}$ and $P_{c_1|\mathbf{z}}$ are distribution functions. Defining $d_\mu(e, \mathbf{z}) = \mu(e, \mathbf{z}) - \mu(\mathbf{z})$, where $\mu(\mathbf{z}) = \lim_{e\to\infty} \mu(e, \mathbf{z})$, the left-hand side of (33) can be written as

$$\begin{aligned}(u(e) - P_{c_0}(e)) * \mu(e, \mathbf{z}) =& \int_0^e (u(\alpha) - P_{c_0}(\alpha))\mu(e - \alpha, \mathbf{z}) d\alpha \\ =&\mu(\mathbf{z}) \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d\alpha \\ &+ \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d_\mu(e - \alpha, \mathbf{z}) d\alpha. \end{aligned} \tag{34}$$

Now we compute the limit of (34) for large e. After some algebra, it can be shown that

$$\lim_{e\to\infty} \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d\alpha = \mathbb{E}\{c_0\}. \tag{35}$$

To compute the limit of the second term in (34), note that, since the limit in (35) is $\mathbb{E}\{c_0\}$, for any $\epsilon > 0$ we can take some q_ϵ such that, for any $e > q_\epsilon$ it holds that $\int_0^e (u(\alpha) - P_{c_0}(\alpha)) d\alpha > \mathbb{E}\{c_0\} - \epsilon$. Also, since $\lim_{e\to\infty} \mu(e, \mathbf{z}) = \mu(\mathbf{z})$, we can take e large enough to get $|\mu(e', \mathbf{z}) - \mu(\mathbf{z})| < \epsilon$ for any $e' > e + q_\epsilon$. Thus,

$$\begin{aligned}&\left| \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d_\mu(e - \alpha, \mathbf{z}) d\alpha \right| \\ &\leq \int_0^{q_\epsilon} (u(\alpha) - P_{c_0}(\alpha)) |d_\mu(e - \alpha, \mathbf{z})| d\alpha \\ &\quad + \int_{q_\epsilon}^e (u(\alpha) - P_{c_0}(\alpha)) |d_\mu(e - \alpha, \mathbf{z})| d\alpha \\ &< \mathbb{E}\{c_0\}\epsilon + \int_{q_\epsilon}^e (u(\alpha) - P_{c_0}(\alpha)) |d_\mu(e - \alpha, \mathbf{z})| d\alpha. \end{aligned} \tag{36}$$

If we can prove that $|d_\mu(e, \mathbf{z})|$ is bounded by some $B_{\mathbf{z}} < \infty$ for any $e \geq 0$ we get

$$\left| \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d_\mu(e - \alpha, \mathbf{z}) d\alpha \right| < (\mathbb{E}\{c_0\} + B_{\mathbf{z}})\epsilon \tag{37}$$

so that

$$\lim_{e\to\infty} \int_0^e (u(\alpha) - P_{c_0}(\alpha)) d_\mu(e - \alpha, \mathbf{z}) d\alpha = 0 \tag{38}$$

and, joining (34), (35) and (38), we arrive at

$$\lim_{e\to\infty} ((u(e) - P_{c_0}(e)) * \mu(e, \mathbf{z})) = \mu(\mathbf{z})\mathbb{E}\{c_0\}. \tag{39}$$

To prove that $|d_\mu(e, \mathbf{z})|$ is bounded, take an arbitrary $\epsilon > 0$. Since $\lim_{e\to\infty} \mu(e, \mathbf{z}) = \mu(\mathbf{z})$, there exists some q such that $|d_\mu(e, \mathbf{z})| < \epsilon$ for any $e > q$. For $e < q$,

$$\begin{aligned}|d_\mu(e, \mathbf{z})| &\leq |\mu(e, \mathbf{z})| + |\mu(\mathbf{z})| \\ &= |\lambda(e) * (p_{c_0|\mathbf{z}}(e) - p_{c_1|\mathbf{z}}(e)| + |\mu(\mathbf{z})| \quad \leq \lambda(q), \end{aligned} \tag{40}$$

where the last inequality uses the fact that $\lambda(e)$ is a nondecreasing function of e. Thus, for any $e \geq 0$, $|d_\mu(e, \mathbf{z})| \geq B = \max\{\epsilon, \lambda(q)\}$, which completes the proof of (39).

The right-hand side of (33) can be analyzed in a similar way: defining $d_h(e) = h(e) - h_\infty$, where $h_\infty = \lim_{e\to\infty} h(e)$, we can write

$$\begin{aligned}&(h(e)u(e)) * (P_{c_0|\mathbf{z}}(e) - P_{c_1|\mathbf{z}}(e)) \\ &= \int_0^e (P_{c_0|\mathbf{z}}(\alpha) - P_{c_1|\mathbf{z}}(\alpha)) h(e - \alpha) d\alpha \\ &= h_\infty \int_0^e (P_{c_0|\mathbf{z}}(\alpha) - P_{c_1|\mathbf{z}}(\alpha)) d\alpha \\ &\quad + \int_0^e (P_{c_0|\mathbf{z}}(\alpha) - P_{c_1|\mathbf{z}}(\alpha)) d_h(e - \alpha) d\alpha. \end{aligned} \tag{41}$$

The same reasoning used to prove (38) can be used to prove that

$$\lim_{e\to\infty} \int_0^e (P_{c_0|\mathbf{z}}(e) - P_{c_1|\mathbf{z}}(e)) d_h(e - \alpha) d\alpha = 0. \tag{42}$$

Defining $\Delta(\mathbf{z}) = \mathbb{E}\{c_1|\mathbf{z}\} - \mathbb{E}\{c_0|\mathbf{z}\}$ and substituting (42) into (41), in the limit we get

$$\begin{aligned}\lim_{e\to\infty}&((h(e)u(e)) * (P_{c_0|\mathbf{z}}(e) - P_{c_1|\mathbf{z}}(e)))\\ &= h_\infty \int_0^e (P_{c_0|\mathbf{z}}(\alpha) - P_{c_1|\mathbf{z}}(\alpha))d\alpha\\ &= h_\infty \Delta(\mathbf{z}). \end{aligned} \tag{43}$$

Combining (33), (39) and (43), we have that $\mu(\mathbf{z})\mathbb{E}\{c_0\} = h_\infty\Delta(\mathbf{z})$, which shows that $\tau = \mu(\mathbf{z})/\Delta(\mathbf{z})$ does not depend on $\mathbf{z}$. Thus

$$\tau\mathbb{E}\{E_0(\mathbf{z})\} = \mathbb{E}\{(xQ(\mathbf{z}) - \Delta(\mathbf{z})\tau)^+\}, \tag{44}$$

which is equivalent to (15). To show that, for $\Delta(\mathbf{z}) > 0$ the solution of (44) is unique, note that the left-hand side is a strictly increasing function of τ while the right-hand side is a non-increasing function, because $d\mathbb{E}\{(xQ(\mathbf{z}) - \Delta(\mathbf{z})\tau)^+\}/d\tau = -\mathbb{E}\{\Delta(\mathbf{z})u(xQ(\mathbf{z}) - \Delta(\mathbf{z})\tau)\}$, which (for $\Delta(\mathbf{z}) > 0$) is always non positive. Since a strictly increasing function intersects with a non-increasing function in at most one single point, the solution is unique.

References

[1] R. Arroyo-Valles, A. G. Marques, and J. Cid-Sueiro, "Optimal selective transmission under energy constraints in sensor networks," *IEEE Trans. Mobile Computing*, vol. 8, no. 11, pp. 1524–1538, Nov. 2009.

[2] R. Arroyo-Valles, A. G. Marques, and J. Cid-Sueiro, *Wireless Sensor Networks*. IN-TECH, 2010, ch. Energy-aware Selective Communications in Sensor Networks.

[3] E. Shih, S.-H. Cho, N. Ickes, R. Min, A. Sinha, A. Wang, and A. Chandrakasan, "Physical layer driven protocol and algorithm design for energy-efficient wireless sensor networks," in *Proc. 7th Annual ACM/IEEE Int'l Conf. on Mobile Computing and Networking (Mobicom 01)*, July 2001.

[4] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, no. 8, pp. 102–114, Aug. 2002.

[5] A. G. Marques, X. Wang, and G. B. Giannakis, "Minimizing transmit-power for coherent communications in wireless sensor networks with finite-rate feedback," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4446–4457, Sep. 2008.

[6] A. Wood and J. Stankovic, "Denial of service in sensor networks," *IEEE Computer*, vol. 35, no. 10, pp. 54–62, Oct. 2002.

[7] V. Shnayder, B. Chen, K. Lorincz, T. Fulford-Jones, and M. Welsh, "Sensor networks for medical care," Division of Engineering and Applied Sciences, Harvard University, Tech. Rep. TR-08-05, 2005.

[8] J. Qiu, Y. Tao, and S. Lu, *Grid and Cooperative Computing*. Springer Berlin / Heidelberg, 2005, vol. 3795, ch. Differentiated Application Independent Data Aggregation in Wireless Sensor Networks, pp. 529–534.

[9] G. Merrett, B. Al-Hashimi, N. White, and N. Harris, "Information managed wireless sensor networks with energy aware nodes," in *Proc. NSTI Nanotechnology Conf. and Trade Show (NanoTech '05)*, May 2005, pp. 367–370.

[10] S. Appadwedula, V. V. Veeravalli, and D. L. Jones, "Energy-efficient detection in sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 693-702, Apr. 2005.

[11] J. Lei, R. Yates, and L. Greenstein, "A generic model for optimizing single-hop transmission policy of replenishable sensors," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 547–551, Feb. 2009.

[12] M. Puterman, *Markov Decision Processes*. Wiley-Interscience, 2005.

[13] Z. Ye, A. Abouzeid, and J. Ai, "Optimal stochastic policies for distributed data aggregation in wireless sensor networks," *IEEE/ACM Trans. Networking*, vol. 17, no. 5, pp. 1494–1507, Oct. 2009.

[14] S. Kianpisheh and N. Charkari, "Dynamic power management for sensor node in WSN using average reward MDP," in *Proc. 4th Int'l Conf. on Wireless Algorithms, Systems, and Applications*, 2009, p. 61.

[15] A. Munir and A. Gordon-Ross, "An MDP-based application oriented optimal policy for wireless sensor networks," in *Proc. 7th IEEE/ACM Int'l Conf. on Hardware/Software Codesign and System Synthesis (CODES+ISSS '09)*, pp. 183–192.

[16] R. Arroyo-Valles, A. G. Marques, J. Vinagre-Díaz, and J. Cid-Sueiro, "A Bayesian decision model for intelligent routing in sensor networks," in *Proc. 3rd IEEE Int'l Symp. on Wireless Comm. Systems (ISWCS '06)*, Sep. 2006.

[17] R. Arroyo-Valles, A. G. Marques, and J. Cid-Sueiro, "Energy-efficient selective forwarding for sensor networks," in *Proc. Workshop on Energy in Wireless Sensor Networks (WEWSN'08)*, in conjunction with DCOSS'08, June 2008.

[18] B. Karp and H. Kung, "Greedy perimeter stateless routing for wireless networks," in *Proc. 6th Annual ACM/IEEE Int'l Conf. on Mobile Computing and Networking (MobiCom 2000)*, Aug. 2000, pp. 243–254.

[19] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *Proc. 33rd Hawaii Int'l Conf. on System Sciences*, 2000, pp. 3005–3014.

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