A Stochastic Approximation Approach to Load Shedding in Power Networks

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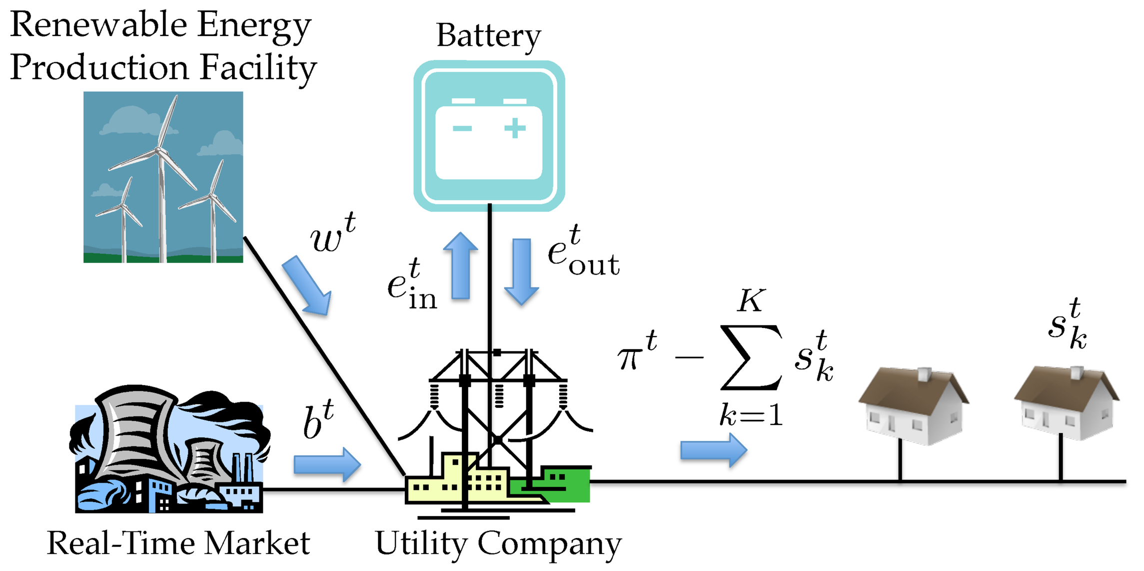
Abstract

A system comprising a utility company serving a set of electricity end-users is considered. The utility company can purchase energy from the wholesale market. It is also connected to a renewable energy production facility, from which it can harvest energy at no cost, and also to a battery for energy storage. Ahead of a scheduling horizon, the utility purchases energy based on forecasted demand and renewable energy production. During online operation, if the renewable energy is not adequate, real-time decisions with respect to user load shedding, energy procurement, and battery charging or discharging need to be made. The problem is cast in a stochastic approximation framework, and is solved online via a dual stochastic subgradient method with low per-slot complexity.

System Model



- Set of users $\{1,\ldots,K\}$; set of slots $\{1,\ldots,T\}$
- Utility company **(load-serving entity, LSE)** can use renewable energy and the battery (at zero cost), and purchased energy from the market
- LSE forecasts user demand and renewable energy ahead of the horizon, and procures energy
- π^t = actual demand − procured energy at slot t
- w^t = produced renewable energy at slot t
- $\pi^t - w^t$ = real-time energy shortage, that has to be provided by **1)** shedded load s_k^t of user k, **2)** energy e_{out}^t from the battery, and **3)** energy b^t purchased in real time at price a^t
- w^t, a^t stationary and ergodic; π^t deterministic
- Cost $J_k\left(\frac{1}{T}\sum_{k=1}^K s_k^t\right)$ for user compensation and fairness; convex and strictly increasing

Battery Dynamics

- r^t = energy stored in the beginning of slot t
- R = battery capacity; $[x]^- = \max\{-x, 0\}$
- LSE charges the battery only if $\pi^t - w^t \le 0$
- e_{in}^t = energy stored in the battery at slot t

$$r^{t+1} = r^t + e_{\text{in}}^t - e_{\text{out}}^t, \quad 0 \le r^t \le R \tag{1}$$

$$0 \le e_{\text{out}}^t \le e_{\text{out}}^{\max}, \quad 0 \le e_{\text{in}}^t \le \min\{e_{\text{in}}^{\max}, [\pi^t - w^t]^-\}$$

Problem Formulation

Minimize

$$\sum_{k=1}^K J_k(\tilde{s}_k) + \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^T a^t b^t \tag{2a}$$

Subject to

Time-average constraints

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^T s_k^t \le \tilde{s}_k, \; k = 1,\ldots,K \tag{2b}$$

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^T e_{\text{out}}^t = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^T e_{\text{in}}^t \tag{2c}$$

Instantaneous constraints $(t = 1, 2, 3, \ldots)$

$$\pi^t - w^t \le \sum_{k=1}^K s_k^t + b^t + e_{\text{out}}^t - e_{\text{in}}^t \tag{2d}$$

$$0 \le e_{\text{in}}^t \le \min\{e_{\text{in}}^{\max}, [\pi^t - w^t]^-\} \tag{2e}$$

$$0 \le e_{\text{out}}^t \le e_{\text{out}}^{\max} \tag{2f}$$

$$0 \le b^t \le b^{\max} \tag{2g}$$

$$0 \le s_k^t \le s_k^{\max}, \; k = 1,\ldots,K \tag{2h}$$

Variables: $\{\tilde{s}_k\}, \{s_k^t\}, \{b^t\}, \{e_{\text{out}}^t\}, \{e_{\text{in}}^t\}$

Stochastic Subgradients

- Multipliers $\boldsymbol{\sigma} = \{\sigma_k\}_{k=1}^K$ for (2b) and ρ for (2c)
- $\tilde{s}_k^*(\sigma_k^t)$, $s_k^{t*}(\boldsymbol{\sigma}^t, \rho^t)$, $e_{\text{out}}^{t*}(\boldsymbol{\sigma}^t, \rho^t)$, and $e_{\text{in}}^{t*}(\boldsymbol{\sigma}^t, \rho^t)$ are minimizers of the Lagrangian function given multipliers $\boldsymbol{\sigma}^t$, ρ^t, and realizations w^t, a^t
- Stochastic iterations [5], [6]; $[x]^+ = \max\{x, 0\}$

$$\sigma_k^{t+1} = \left[\sigma_k^t - \mu_\sigma\left(\tilde{s}_k^*(\sigma_k^t) - s_k^{t*}(\boldsymbol{\sigma}^t, \rho^t)\right)\right]^+ \tag{3}$$

$$\rho^{t+1} = \left[\rho^t - \mu_\rho\left(e_{\text{in}}^{t*}(\boldsymbol{\sigma}^t, \rho^t) - e_{\text{out}}^{t*}(\boldsymbol{\sigma}^t, \rho^t)\right)\right]^+ \tag{4}$$

Alternative version, with constant C dependent on R:

$$\rho^{t+1} = [C - \mu_\rho r^{t+1}]^+ \tag{5}$$

Online Load Shedding

For each k, initialize σ_k^1 to a small random number.
for $t = 1, \ldots, T$ **do**

[s1] Set $\tilde{s}_k^*(\sigma_k^t) \in \arg\min\{J_k(s) - \sigma_k s\}$.

[s2.0] Set the value of ρ^t using (5).

[s2.1] If $\pi^t - w^t \le 0$, then there is instantaneous energy surplus, which is used to charge the battery. Set the variables as follows and go to **[s3]**.

$$e_{\text{in}}^{t*}(\boldsymbol{\sigma}^t, \rho^t) = \min\{e_{\text{in}}^{\max}, w^t - \pi^t, R - r^t\}$$
$$e_{\text{out}}^{t*}(\boldsymbol{\sigma}^t, \rho^t) = 0, \; b^{t*}(\boldsymbol{\sigma}^t, \rho^t) = 0, \; s_k^{t*}(\boldsymbol{\sigma}^t, \rho^t) = 0.$$

[s2.2] If $\pi^t - w^t > 0$, then there is instantaneous energy shortage. Set $e_{\text{in}}^{t*}(\boldsymbol{\sigma}^t, \rho^t) = 0$, and set $e_{\text{out}}^{t*}(\boldsymbol{\sigma}^t, \rho^t)$, $b^{t*}(\boldsymbol{\sigma}^t, \rho^t)$ and $s_k^{t*}(\boldsymbol{\sigma}^t, \rho^t)$ as the solution of the following linear program with $\tilde{e}_{\text{out}}^{\max} := \min\{e_{\text{out}}^{\max}, r^t\}$. If multiple solutions exist, pick one at random. Go to **[s3]**.

$$\min_{\substack{\{0\le s_k^t \le s_k^{\max}, 0 \le b^t \le b^{\max},\\ 0 \le e_{\text{out}}^t \le \tilde{e}_{\text{out}}^{\max}\}}} \quad a^t b^t + \rho^t e_{\text{out}}^t + \sum_{k=1}^K \sigma_k^t s_k^t$$
$$\text{subj. to} \quad \pi^t - w^t = b^t + e_{\text{out}}^t + \sum_{k=1}^K s_k^t.$$

[s3] Using the outputs of steps **[s1]** and **[s2]**, update the battery r^t via (1) and σ_k^t via (3).

end for

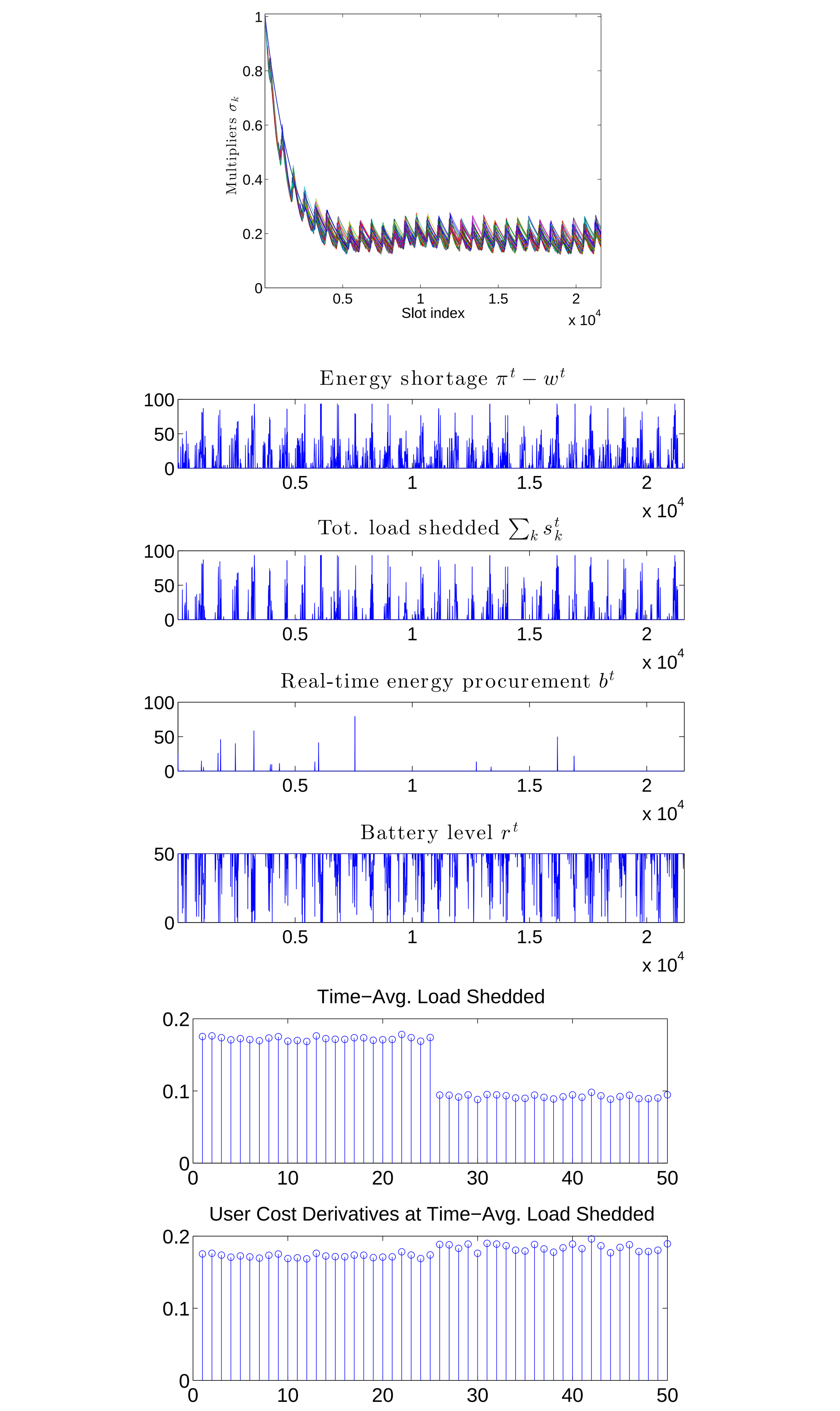
Remarks

- Constraint (2c) relaxes the battery dynamics (1)
- If (1) were included, the problem would be a DP; the relaxation renders the problem convex
- Two modifications of the stochastic subgradient algorithm proposed: **1)** update (5) replaces (4), and **2)** battery dynamics accounted for in **[s3]**
- Online algorithm guarantees asymptotic feasibility and bounded optimality loss

Numerical Test Setup

- $K = 50$ residential users; two classes: $J_k(s_k) = 0.5 s_k^2$ $(1 \le k \le 25)$; $J_k(s_k) = s_k^2$ $(26 \le k \le 50)$
- $T = 21600$ 2-minute intervals (30 days)
- π^t peaks at 300 kW in the evening; $R = 50$ kWh
- a^t uniform in (0,5) cents/kW
- Wind power capacity 130kWh; w^t follows [7]

Results



x-axis: User index. Avg. load shedded: $\bar{s}_k = \frac{1}{T}\sum_{t=1}^T s_k^t$. User-cost derivatives: $J'(\bar{s}_k)$.

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