Hierarchical Spectrum Sharing Using Interference Tweets

Emiliano Dall'Anese*, Antonio G. Marques†, and Georgios B. Giannakis*
*University of Minnesota, Dept. of ECE and Digital Technology Center, Minneapolis, USA
†King Juan Carlos University, Dept. of Signal Theory and Comms., Madrid, Spain

***Abstract*—Spectrum sensing schemes focus on detecting active transmitters, but not incumbent passive receivers, which have nevertheless to be protected from excessive interference. The present paper advocates the notion of *receiver map* as a tool for unveiling areas where licensed receivers are likely to reside, and for limiting the interference accordingly. Receiver maps are tracked using a Bayesian approach, based on a one-bit message - here referred to as "*interference tweet*" - broadcasted by the primary receiver whenever a communication disruption occurs due to interference. The resultant maps are utilized in a cross-layer resource allocation scheme, which is designed to optimize the performance of an underlay multi-hop cognitive radio network under long-term probability of interference constraints. Although non-convex, the problem exhibits zero duality gap, and it is optimally solved using a Lagrangian dual approach.**

Keywords*—Cognitive radios, cross-layer design, receiver localization, recursive Bayesian estimation.

I. Introduction

In the hierarchical spectrum access setup, underlay cognitive radios (CRs) can reuse the frequency bands licensed to a primary user (PU) system, provided ongoing primary communications are not overly disrupted [1]. Instrumental to controlling interference is knowledge of the propagation gains between secondary users (SUs) and PUs. However, since the primary system has generally no incentive to exchange synchronization and channel training signals with SUs, these gains are hardly possible to acquire in practice. In lieu of SU-to-PU channel gains, the distribution of these channels can be used to limit the interference inflicted to PUs by means of either probabilistic or average constraints [2]–[6].

Either way, knowledge of the PU receivers' locations is necessary to evaluate the mean SU-to-PU propagation gains. Since spectrum sensing schemes cannot detect and localize "passive" PU *receivers*, it follows that even the mean SU-to-PU propagation gains (and hence the channel distribution) are in general *uncertain*. The present paper advocates the notion of *receiver map* as a tool for unveiling areas where PU receivers reside, with the objective of limiting the interference inflicted to those locations. These maps are tracked using a recursive Bayesian estimator, which is based on a 1-bit message - reminiscent of modern real-time social messaging systems, thus the term "interference tweet" - broadcasted by the PU system whenever the interference inflicted to one or more PU exceeds a tolerable level.

In the *mainstream* CR literature, SUs are de facto envisioned to gain access to licensed frequencies without requiring any modification in the operation of primary systems. Requiring the PU system to broadcast one bit when disruptive interference occurs, involves a slight modification in the operation of the primary (as opposed to the significant changes in the protocols required in the case where channel training signals are exchanged). However, it will be shown that substantial improvements in spectrum (re)use efficiency can be obtained with this minimal system interplay, thus reaping off the benefits offered by the CR technology to the full extent. In setups where the broadcasting of tweets is not feasible, the secondary network can still estimate the receiver map by e.g., overhearing retransmission requests of the PUs [5], [7].

Based on the receiver maps, a cross-layer resource allocation (RA) scheme is formulated, where PUs are protected by means of long-term probability-of-interference constraints. Access among SUs is assumed orthogonal, and the resources at the transport, network, link and physical layers are adapted to the *time-varying* SU-to-SU *channels and* the receiver *maps*. Although nonconvex, the formulated problem has zero duality gap, and it is optimally solved using a Lagrangian dual approach. Taking advantage of the problem separability across SUs in the dual domain, computationally-affordable optimal solvers for transmit-powers, scheduling and routing variables, as well as exogenous traffic rates are developed[1].

II. Modeling and Problem Formulation

Consider a multi-hop SU network comprising M nodes $\{U_m\}_{m=1}^M$ deployed over an area $\mathcal{A} \subset \mathbb{R}^2$. SUs share a flat-fading frequency band with an incumbent PU system in an underlay mode [1]. In this setup, the channel state information (CSI) available to the SU system is heterogeneous, in the sense that CSI per link may be more or less accurate depending on whether PUs or SUs are involved [4], [5], [8]. Provided bandwidth is available for the SUs to transmit, the SU-to-SU channels can be readily acquired by employing conventional training-based channel estimators. For this reason, the state of the SU-to-SU channels is assumed known. The instantaneous gain of link $U_m \rightarrow U_n$ is denoted as $g_{m,n}$, and it is given by the squared magnitude of the small-scale fading realization scaled by the average signal-to-interferference-plus-noise ratio (SINR).

This work was supported by the QNRF grant NPRP 09-341-2-128.

[1]*Notation:* $\mathbb{E}_{\mathbf{g}}[\cdot]$ denotes expectation with respect to the random process $\mathbf{g}$; $\Pr\{A\}$ the probability of event A; x^* the optimal value of x; $\mathbb{1}_{\{\cdot\}}$ the indicator function ($\mathbb{1}_{\{x\}} = 1$ if x is true, and zero otherwise); $[x]_+$ the projection of the scalar x onto the non-negative orthant; and, $[x]_a^b := \min\{\max\{x, a\}, b\}$ the projection of the scalar x onto $[a, b]$. Given a function $V(x)$, $\dot{V}(x)$ denotes its derivative, and $(V)^{-1}(x)$ the inverse function, provided it exists. Finally, $\wedge$ denotes the logical "and" operator.

Suppose now that PU transmitters communicate with Q PU *receivers* geolocated at $\{\mathbf{x}^{(q)} \in \mathcal{A}\}_{q=1}^{Q}$. Let $h_{m,\mathbf{x}^{(q)}}$ denote the instantaneous channel gain between U_m and PU receiver q. Since PUs have generally no incentive to use primary spectral resources to exchange synchronization and channel training signals with SUs [1], training-based channel estimation cannot be employed at the SU end to acquire $\{h_{m,\mathbf{x}^{(q)}}\}$. Thus, even though the average link gain can be obtained based on locations $\{\mathbf{x}^{(q)} \in \mathcal{A}\}_{q=1}^{Q}$, the instantaneous value of the primary link is *uncertain* due to random fast fading effects. Hereafter, it is assumed that only the joint distribution of processes $\{h_{m,\mathbf{x}^{(q)}}\}$, denoted by $\phi_h(\{h_{m,\mathbf{x}^{(q)}}\})$, is known to the SU network. With this information, and knowing the maximum instantaneous interference power I tolerable by the PUs, SUs can determine the interference probabilities at each location $\mathbf{x}^{(q)}$. Let $z_{\mathbf{x}}^{(q)}$ be a binary variable taking the value 1 if PU receiver q is located at $\mathbf{x} \in \mathcal{A}$. Further, consider discretizing the PU coverage region into a set of grid points $\mathcal{G} := \{\mathbf{x}_g\}$ representing *potential* locations for the PU receivers. In lieu of $\{z_{\mathbf{x}}^{(q)}\}$, the idea here is to use the set of probabilities $\beta_{\mathbf{x}}^{(q)} := \Pr\{z_{\mathbf{x}}^{(q)} = 1\}$, $\forall\, \mathbf{x} \in \mathcal{G}$, to identify areas where a PU receiver q is more likely to reside. To this end, the following is assumed.

(as1) Processes $\{g_{m,n}, h_{m,\mathbf{x}^{(q)}}\}$ are mutually independent.
(as2) Processes $z_{\mathbf{x}}^{(q)}$ and $z_{\mathbf{x}}^{(v)}$, $q \neq v$, are independent.

Assumption *(as2)* presupposes that each PU receiver has its own mobility pattern, while *(as1)* implies that the uncertain component of $\{h_{m,\mathbf{x}^{(q)}}\}$ is spatially uncorrelated. In the following, sets $\mathbf{g} := \{g_{m,n}\}$ and $\mathbf{s} := \{\phi_h\} \cup \{\beta_{\mathbf{x}}^{(q)}\}$ collect the available secondary CSI, and the statistical primary state information (PSI), respectively.

RA under primary state uncertainty. Application-level data packets are generated exogenously at the SUs, and are routed throughout the network to the intended destination(s). Flows are indexed by k, and the packet arrivals at U_m per flow k are modeled by a stationary stochastic process with mean $a_m^k \geq 0$. Let $r_{m,n}^k(\mathbf{g}, \mathbf{s}) \geq 0$ be the instantaneous rate used to route packets of flow k through link $U_m \to U_n$. Suppose that SUs are equipped with buffers to store exogenous and endogenous packets, and let $b_m^k[t]$ denote the queue length at node m for flow k at time t. Queues are deemed stable if $\lim_{t\to+\infty}(1/t)\sum_{\tau=1}^{t}\mathbb{E}[b_m^k[\tau]] < \infty$. Thus, to have stable queues, exogenous and endogenous rates must satisfy the following necessary condition per SU m and flow k:

$$a_m^k + \sum_{n\in\mathcal{N}_m} \mathbb{E}_{\mathbf{g},\mathbf{s}}\left[r_{n,m}^k(\mathbf{g},\mathbf{s})\right] \leq \sum_{n\in\mathcal{N}_m} \mathbb{E}_{\mathbf{g},\mathbf{s}}\left[r_{m,n}^k(\mathbf{g},\mathbf{s})\right]. \quad (1)$$

As for the medium access layer, define a binary scheduling variable $w_{m,n}$ taking the value 1 if U_m is scheduled to transmit to its neighbor U_n, and 0 otherwise. Secondary transmissions are assumed orthogonal in time and space. Orthogonal access is adopted by a gamut of wireless systems because of its low-complexity implementation. On the other hand, it enables a (nearly) optimal network operation under moderate-to-strong interference transmission scenarios. Assuming that one SU link is scheduled per time slot, it follows that

$$\sum_{(m,n)\in\mathcal{E}} w_{m,n}(\mathbf{g},\mathbf{s}) \leq 1 \quad (2)$$

where $\mathcal{E} := \{(m,n) : n \in \mathcal{N}_m, m = 1,\dots,M\}$ represents the set of SU-to-SU links.

At the physical layer, the rate-power coupling is modeled using Shannon's capacity formula $C_{m,n}(\mathbf{g}, p_{m,n}) = W\log(1+p_{m,n}g_{m,n}/\kappa_{m,n})$, where $\kappa_{m,n}$ represents the coding scheme-dependent SINR gap, and W is the bandwidth of the primary channel that is to be (re-)used. Let $\bar{p}_m$ denote the average transmit-power of U_m, which can be expressed as $\bar{p}_m := \mathbb{E}_{\mathbf{g},\mathbf{s}}\left[\sum_{n\in\mathcal{N}_m} w_{m,n}(\mathbf{g},\mathbf{s})p_{m,n}(\mathbf{g},\mathbf{s})\right]$. Two different constraints are imposed on the transmit-powers. First, due to spectrum mask specifications, the instantaneous power $p_{m,n}$ cannot exceed a pre-defined limit $p_m^{\max}$. Secondly, a cap $\bar{p}_m^{\max}$ may be specified for $\bar{p}_m$.

To account for the interference inflicted to the PU system [1], define the binary variable $i^{(q)}(\{p_{m,n}\}, \mathbf{s})$ as

$$i^{(q)}(\{p_{m,n}\}, \mathbf{s}) := \sum_{\mathbf{x}\in\mathcal{G}} \mathbb{1}_{\{\sum_{(m,n)} w_{m,n}p_{m,n}h_{m,\mathbf{x}^{(q)}} > I\}} z_{\mathbf{x}}^{(q)}. \quad (3)$$

Since $z_{\mathbf{x}}^{(q)}$ pinpoints the location of PU receiver q, variable $i^{(q)}(\{p_{m,n}\}_{m,n}, \mathbf{s})$ clearly indicates whether or not excessive instantaneous interference is inflicted to PU receiver q. Further, define the binary random variable

$$i(\{p_{m,n}\}_{m,n}, \mathbf{s}) := 1 - \prod_{q=1}^{Q}\left(1 - i^{(q)}(\{p_{m,n}\}, \mathbf{s})\right), \quad (4)$$

which equals 1 if one or more PU receivers are interfered. Since $w_{m,n}(\mathbf{g},\mathbf{s}) \in \{0,1\}$, and at most one secondary link is active per time slot, $i(\{p_{m,n}(\mathbf{g},\mathbf{s})\}, \mathbf{s})$ can be rewritten as $i(\{p_{m,n}(\mathbf{g},\mathbf{s})\}, \mathbf{s}) = \sum_{(m,n)\in\mathcal{E}} w_{m,n}(\mathbf{g},\mathbf{s}) i_{m,n}(p_{m,n}(\mathbf{g},\mathbf{s}), \mathbf{s})$, where

$$i_{m,n}(p_{m,n}, \mathbf{s}) := 1 - \prod_{q=1}^{Q}\left(1 - \sum_{\mathbf{x}\in\mathcal{G}} \mathbb{1}_{\{p_{m,n}h_{m,\mathbf{x}^{(q)}} > I\}} z_{\mathbf{x}}^{(q)}\right).$$

With $i^{\max} \in (0,1)$ denoting the maximum (long-term) rate of interference, the following constraint must hold

$$\mathbb{E}_{\mathbf{g},\mathbf{s}}\Big[\sum_{(m,n)\in\mathcal{E}} w_{m,n}(\mathbf{g},\mathbf{s}) i_{m,n}(p_{m,n}(\mathbf{g},\mathbf{s}), \mathbf{s})\Big] \leq i^{\max}. \quad (5)$$

The SU network operates in a time-slotted setup, where the duration of each slot (indexed by t) corresponds to the coherence time of the small-scale fading process. The PSI $\mathbf{s}$ may also vary (due to, e.g., PU mobility), but at a larger time scale. Thus, since $\{r_{n,m}^k, w_{m,n}, p_{m,n}\}$ are computed based on $\mathbf{g}$ and $\mathbf{s}$, it follows that $\{r_{n,m}^k, w_{m,n}, p_{m,n}\}$ vary too. Henceforth, $\mathbf{g}$, $\mathbf{s}$, $r_{n,m}^k(\mathbf{g},\mathbf{s})$, $w_{m,n}(\mathbf{g},\mathbf{s})$, and $p_{m,n}(\mathbf{g},\mathbf{s})$ will be replaced by $\mathbf{g}[t]$, $\mathbf{s}[t]$, $r_{n,m}^k[t]$, $w_{m,n}[t]$, and $p_{m,n}[t]$, whenever time dependence is to be stressed.

Let $V_m^k(a_m^k)$ denote a concave, non-decreasing, utility function quantifying the reward associated with the exogenous rate a_m^k, and $J_m(\bar{p}_m)$ be a convex, increasing, function representing the cost incurred by U_m when an average transmit-power $\bar{p}_m$ is used. Further, assume that $V_m^k(a_m^k)$ and $J_m(\bar{p}_m)$ are differentiable.

Based on the preceding discussion and with $\mathbf{y} := \{a_m^k, r_{n,m}^k(\mathbf{g},\mathbf{s}), w_{m,n}(\mathbf{g},\mathbf{s}), p_{m,n}(\mathbf{g},\mathbf{s}), \forall m,n \in \mathcal{N}_m, \mathbf{g},\mathbf{s}\}$ accounting for all design variables (resources), the optimal cross-layer RA problem is formulated as follows:

$$\mathsf{P}^* := \max_{\mathbf{y}} \sum_{m,k} V_m^k(a_m^k) - \sum_m J_m(\bar{p}_m) \tag{6a}$$

$$\text{subject to}: (1),\ (2),\ (5),\ \text{and} \tag{6b}$$

$$w_{m,n}(\mathbf{g},\mathbf{s}) C_{m,n}(\mathbf{g}, p_{m,n}(\mathbf{g},\mathbf{s})) \geq \sum_k r_{m,n}^k(\mathbf{g},\mathbf{s}) \tag{6c}$$

$$\mathbb{E}_{\mathbf{g},\mathbf{s}}\left[\sum_{n\in\mathcal{N}_m} w_{m,n}(\mathbf{g},\mathbf{s}) p_{m,n}(\mathbf{g},\mathbf{s})\right] \leq \bar{p}_m \tag{6d}$$

$$w_{m,n}(\mathbf{g},\mathbf{s}) \in \{0,1\},\ a_m^{k,\min} \leq a_m^k \leq a_m^{k,\max},\ 0 \leq r_{m,n}^k \tag{6e}$$

$$0 \leq p_{m,n} \leq p_m^{\max},\ 0 \leq \bar{p}_{m,n} \leq \bar{p}_m^{\max} \tag{6f}$$

where (6c) guarantees that the rate at the network layer cannot exceed the rate provided by the physical and link layers, and (6d)-(6f) set limits (bounds) on the values the resources can take. Unfortunately, (6) is a challenging nonconvex problem. Specifically, non-convexity is due to: *i)* $\{w_{m,n}(\mathbf{g},\mathbf{s})\}$ are binary; *ii)* monomials $w_{m,n}p_{m,n}$ and $w_{m,n}C_{m,n}(\mathbf{g},p_{m,n})$ are not *jointly* convex in $w_{m,n}$ and $p_{m,n}$; and, *iii)* the interference constraint (5) is nonconvex. Despite these difficulties, the *optimal* solution to (6) will be obtained next.

III. Optimal Cross-layer RA

Consider first relaxing the binary scheduling constraints $w_{m,n}(\mathbf{g},\mathbf{s}) \in \{0,1\}$ as $w_{m,n}(\mathbf{g},\mathbf{s}) \in [0,1]$. Since constraints (2), (6c), (6d), and (5) are linear in $w_{m,n}(\mathbf{g},\mathbf{s})$, it can be rigorously shown that the optimal $\{w_{m,n}^*\}$ in the relaxed problem coincide with those in (6). Next, to cope with the non-convexity of the monomial $w_{m,n}(\mathbf{g},\mathbf{s})p_{m,n}(\mathbf{g},\mathbf{s})$ and function $w_{m,n}(\mathbf{g},\mathbf{s})C_{m,n}(\mathbf{g},p_{m,n}(\mathbf{g},\mathbf{s}))$, consider introducing the auxiliary variables $\tilde{p}_{m,n}(\mathbf{g},\mathbf{s}) := w_{m,n}(\mathbf{g},\mathbf{s})p_{m,n}(\mathbf{g},\mathbf{s})$, $(m,n) \in \mathcal{E}$. It can be verified that the Hessian of the function $w_{m,n}(\mathbf{g},\mathbf{s})C_{m,n}(\mathbf{g},\tilde{p}_{m,n}(\mathbf{g},\mathbf{s})/w_{m,n}(\mathbf{g},\mathbf{s}))$ is semi-negative definite, and thus the resultant surrogate constraint is convex. Unfortunately, there is no immediate way to address the non-convexity of (5). Nevertheless, one can leverage the results of [9, Thm. 1] to show that the duality gap is *zero*, and adopt a Lagrange dual approach *without* loss of optimality.

Consider then dualizing the average constraints, namely, (1), (5), and (6d), and let $\{\lambda_m^k\}$, θ, and $\{\pi_m\}$ denote the corresponding Lagrange multipliers. Assuming that the optimal multipliers $\mathbf{d}^* := \{\{\lambda_m^{k*}\}, \{\pi_m^*\}, \theta^*\}$ are available, the optimal primal variables can be computed as follows[2].

Proposition 1. *The optimal average transmit-power $\bar{p}_m^*$ of node U_m is found as the solution of the following scalar convex program*

$$\bar{p}_m^*(\pi_m^*) := \underset{0\leq\bar{p}_m\leq\bar{p}_m^{\max}}{\arg\max} \; -J_m(\bar{p}_m) + \pi_m^*\bar{p}_m\,. \tag{7}$$

If $\dot{J}_m(\bar{p}_m)$ is invertible, then $\bar{p}_m^*(\pi_m^*) = \left[(\dot{J}_m)^{-1}(\pi_m^*)\right]_0^{\bar{p}_m^{\max}}$.

This result can be obtained by maximizing the Lagrangian with respect to $\{\bar{p}_m\}$, and exploiting its decomposability in M sub-problems (one per variable $\bar{p}_m$) [11]. Following similar steps, the average exogenous rates are found next.

[2]Proofs of propositions are omitted due to space limitations; see [10].

Proposition 2. *Given the optimal dual variables $\{\lambda_m^{k*}\}$, the optimal exogenous rates $\{a_m^{k*}\}$ are given by*

$$a_m^{k*}(\lambda_m^{k*}) = \underset{a_m^{k,min}\leq a\leq a_m^{k,max}}{\arg\max} \; V_m^k(a) - \lambda_m^{k*}a\,. \tag{8}$$

When the inverse of $\dot{V}_m^k(a)$ exists, a_m^{k*} can be obtained in closed form as $a_m^{k*}(\lambda_m^{k*}) = \left[(\dot{V}_m^k)^{-1}(\lambda_m^{k*})\right]_{a_m^{k,min}}^{a_m^{k,max}}$.

Define the coefficients $\lambda_{m,n}^{k*} := \lambda_m^{k*} - \lambda_n^{k*}$ per link $(m,n) \in \mathcal{E}$ and flow k, along with the functional

$$\varphi_{m,n}(\mathbf{g}[t], p, \mathbf{d}^*) := \Big[\lambda_{m,n}^* C_{m,n}(\mathbf{g}[t], p) - \pi_m^* p - \theta^* \mathbb{E}_{\mathbf{s}[t]}\left[i_{m,n}(p,\mathbf{s}[t])\right]\Big]_+ \tag{9}$$

where $\lambda_{m,n}^* := \max_k\{\lambda_{m,n}^{k*}\}$, and $\{\mathbf{g}[t], \mathbf{s}[t]\}$ are the realizations of $\{\mathbf{g},\mathbf{s}\}$ at slot t. Notice that, using *(as1)–(as2)*, $\mathbb{E}_{\mathbf{s}[t]}\left[i_{m,m}(p,\mathbf{s}[t])\right]$ can be simplified as $\mathbb{E}_{\mathbf{s}[t]}\left[i_{m,n}(p,\mathbf{s}[t])\right] = 1 - \prod_{q=1}^{Q}\left(1 - \sum_{\mathbf{x}\in\mathcal{G}} \iota_{m,\mathbf{x}^{(q)}}(p,\mathbf{s}[t])\beta_{\mathbf{x}}^{(q)}\right)$, where the quantity $\iota_{m,\mathbf{x}^{(q)}}(p,\mathbf{s}[t]) := \Pr\left\{p\, h_{m,\mathbf{x}^{(q)}} > I\right\}$ can be computed once the distribution of $h_{m,\mathbf{x}^{(q)}}$ is known. Using (9), the remaining design variables are found as specified next.

Proposition 3. *Given realizations $\mathbf{g}[t]$ and $\mathbf{s}[t]$, the optimal $\{w_{m,m}^*[t]\}$ and $\{p_m^*[t]\}$ are given by*

$$p_{m,n}^*[t] := \left[\arg\max_p \; \varphi_{m,n}(\mathbf{g}[t], p, \mathbf{d}^*))\right]_0^{p_m^{\max}} \tag{10}$$

$$w_{m,n}^*[t] := \mathbb{1}_{\{(m,n)=\arg\max_{(i,j)\in\mathcal{E}} \varphi_{m,n}(\mathbf{g}[t], p_{m,n}^*[t], \mathbf{d}^*)\}}\,. \tag{11}$$

Proposition 4. *Per link $(m,n) \in \mathcal{E}$, define the set $\mathcal{R}_{m,n}[t] := \{j : j = \arg\max_k\{\lambda_{m,n}^{k*}\} \wedge \lambda_{m,n}^{j*} \geq 0\}$. The optimal instantaneous routing variables $\{r_{m,n}^{k*}[t]\}$ satisfy then the following two conditions:*

r1) if $k \notin \mathcal{R}_{m,n}[t]$, then $r_{m,n}^{k*}[t] = 0$; and,

r2) if $|\mathcal{R}_{m,n}[t]| \geq 1$, it follows that $\sum_{k\in\mathcal{R}_{m,n}[t]} r_{m,n}^{k*}[t] = w_{m,m}^*[t]\, C_{m,n}(\mathbf{g}[t], p_{m,n}^*[t])$.

When $|\mathcal{R}_{m,n}[t]| = 1$, one has the "winner-takes-all" solution $r_{m,n}^{k*}[t] = \mathbb{1}_{\{k\in\mathcal{R}_{m,n}[t]\}} w_{m,m}^*[t]\, C_{m,n}(\mathbf{g}[t], p_{m,n}^*[t])$.

For many fading distributions, (10) turns out to be non-convex. However, since only one scalar variable is involved, $p_{m,n}^*[t]$ can be efficiently found.

Estimation of the optimum Lagrange multipliers. To find the optimal dual variables $\{\lambda_m^{k*}, \pi_m^*, \theta^*\}$, a stochastic approximation method is adopted [12]. Stochastic approximation methods incur a computational complexity that is much lower than that of their off-line counterparts, and cope with non-stationary channels and dynamic PU activities. With $\mu_\lambda > 0$, $\mu_\pi > 0$, and $\mu_\theta > 0$ denoting constant stepsizes, the following iterations yield the desired multipliers $\forall\, t$:

$$\lambda_m^k[t+1] = \Big[\lambda_m^k[t] + \mu_\lambda\Big(a_m^{k*}(\lambda_m^k[t]) + \sum_{n\in\mathcal{N}_m}(r_{n,m}^{k*}[t] - r_{m,n}^{k*}[t])\Big)\Big]_+$$

$$\pi_m[t+1] = \Big[\pi_m[t] - \mu_\pi\Big(\bar{p}_m^*(\pi_m[t]) - \sum_{n\in\mathcal{N}_m} w_{m,n}^*[t] p_{m,n}^*[t]\Big)\Big]_+$$

$$\theta[t+1] = \Big[\theta[t] - \mu_\theta\Big(i^{\max} - i[t]\Big)\Big]_+ \tag{12}$$

where $i[t] = 1$ if an interference tweet is broadcasted at time t, and 0 otherwise.

Updates in (12) form an *unbiased* stochastic subgradient of the dual function of (6), and they are *bounded* [12]. Define $\mu := \max\{\mu_\lambda, \mu_\pi, \mu_\theta\}$; $\mathsf{P}[t] := \frac{1}{t}\sum_{l=1}^{t}\sum_{m,k} V_m^k(a_m^{k*}[t]) + \sum_m J_m(\bar{p}_m^*[t])$; and, $\bar{i}[t] := \frac{1}{t}\sum_{l=1}^{t} i[l]$. Then, it can be shown that, as $t \to \infty$, the following holds with probability one: *i)* $\bar{i}[t] = i^{\max}$; and, *ii)* $\mathsf{P}[t] \geq \mathsf{P}^* - \delta(\mu)$, where $\delta(\mu) \to 0$ as $\mu \to 0$. A proof of this result is not presented here due to space limitations, but it relies on the convergence of stochastic (epsilon) subgradient methods and can be obtained following the lines of [12]. The key to prove *i)* is to show that the stochastic multipliers are bounded. This can be readily used to show the asymptotic feasibility of the sample averages of the stochastic subgradients, i.e., of the update terms in (12). The proof for *ii)* is a bit more intricate and leverages properties of the dual function, the convexity of the objective function in (6a), and the bounds on the stochastic updates. It turns out that the loss of optimality $\delta(\mu)$ is linear w.r.t. both μ and G, which represents an upperbound on the expected squared norm of the stochastic subgradient. Clearly, this implies that $\delta(\mu) \to 0$ as $\mu \to 0$.

IV. Receiver map estimation

The SUs rely on the interference probabilities $\{\iota_{m,\mathbf{x}}\}$ and PU receivers' spatial distribution $\{\beta_{\mathbf{x}}^{(q)}\}$ to schedule SU transmissions and limit their powers based on the expected probability of interference $\mathbb{E}_{\mathbf{s}[t]}\left[i_m(p, \mathbf{s}[t])\right]$ [cf. (9)]. Once $\mathcal{G}$ is chosen, $\{\iota_{m,\mathbf{x}}\}$ can be computed as a function of the powers $\{p_{m,n}\}$. The aim here is to develop an online Bayesian estimator for $\{\beta_{\mathbf{x}}^{(q)}\}$ based on the following assumption.

(as3) the PU system broadcasts the message $i^{(q)}[t] = 1$ to notify that the event $p_{m^*,n^*}^* h_{m^*,\mathbf{x}^{(q)}} > I$ occurred.

Just *one* bit is sufficient to notify the SU system that the instantaneous interference inflicted to one PU receiver exceeded the safety level I. The interference tweet can be sent by either the primary network controller (requiring a few additional bits to indicate the PU that was interferred), or, by the interferred PU receivers (and the PU identifier is included as usual in the packet header). Similar modeling assumptions were made in, e.g., [5] and [7] (see also references therein), where the PU's Automatic Repeat-reQuests (ARQs) are assumed to be either exchanged or eavesdropped by the SU transceivers. With the overheard ARQs, the SUs can evaluate the outage rates of ongoing PU communications, and adjust their transmit-powers accordingly [5]. In lieu of outage rates, PU receiver locations may be estimated. However, localization based on RSS measurements over a single ARQ packet is challenging because of, e.g., PU mobility and fast time-varying fading. A more conservative (but suboptimal) approach that bypasses the need to know PU receiver locations is to guarantee that the interference does not exceed a prescribed level at any boundary point of the PU transmitters' coverage region [8], which can be estimated during the sensing phase. This amounts to arranging $Q = |\mathcal{G}|$ grid points in the boundary of the coverage region, and setting $\beta_{\mathbf{x}}^{(q)} = 1$ for all $q = 1, \ldots, Q$.

To account for PU mobility, $z_{\mathbf{x}}^{(p)}[t]$ is modeled as a first-order (spatiotemporal) Markov process characterized by the transition probabilities $\phi_{\mathbf{x},\mathbf{x}'}^{(q)}[t] := \Pr\{z_{\mathbf{x}}^{f,(q)}[t] = 1 | z_{\mathbf{x}'}^{(q)}[t-1] = 1\}$. Such transition probabilities are assumed to be non-zero only if $\mathbf{x}' \in \mathcal{G}_{\mathbf{x}}$, where the set $\mathcal{G}_{\mathbf{x}}$ contains $\mathbf{x}$ and its neighboring grid points. Collect in the set $\mathcal{I}_t^{(q)} := \{i^{(q)}[\tau], \tau = 1, \ldots, t\}$ the interference notifications up to time slot t, and define further the sets $\mathcal{H}_t^{(q)} := \mathcal{I}_t^{(q)} \cup \{p_{m,n}^*[\tau], w_{m,n}^*[\tau], \tau = 1, \ldots, t\}$ and $\tilde{\mathcal{H}}_t^{(q)} := \mathcal{H}_{t-1}^{(q)} \cup \{p_{m,n}^*[t], w_{m,n}^*[t], \ \forall (m,n)\}$. Since the elements of $\mathcal{I}_t^{(q)}$ constitute the observed states of a Hidden Markov Model (HMM), a recursive Bayes estimator can be efficiently implemented to acquire (and track) the posterior probability mass function of $\{z_{\mathbf{x}}^{(q)}\}_{\mathbf{x} \in \mathcal{G}}$. To this end, let $\hat{\beta}_{\mathbf{x}}^{(q)}[t|t-1] := \Pr\{z_{\mathbf{x}}^{(q)}[t] = 1 | \mathcal{H}_{t-1}^{(q)}\}$ and $\hat{\beta}_{\mathbf{x}}^{(q)}[t|t] := \Pr\{z_{\mathbf{x}}^{(q)}[t] = 1 | \mathcal{H}_t^{(q)}\}$ denote the instantaneous beliefs given $\mathcal{H}_{t-1}^{(q)}$ and $\mathcal{H}_t^{(q)}$, respectively. Thus, the receiver maps can be recursively updated by performing the following *prediction* and *correction* steps per grid point $\mathbf{x}$ and PU receiver q (see e.g., [13]).

$$\hat{\beta}_{\mathbf{x}}^{(q)}[t|t-1] = \sum_{\mathbf{x}' \in \mathcal{G}_{\mathbf{x}}} \phi_{\mathbf{x},\mathbf{x}'}^{(q)}[t] \hat{\beta}_{\mathbf{x}}^{(q)}[t-1|t-1] \tag{13}$$

$$\hat{\beta}_{\mathbf{x}}^{(q)}[t|t] = \frac{\Pr\{i^{(q)}[t] = o | z_{\mathbf{x}}^{(q)}[t] = 1, \tilde{\mathcal{H}}_t^{(q)}\} \hat{\beta}_{\mathbf{x}}^{(q)}[t|t-1]}{\Pr\{i^{(q)}[t] = o | \tilde{\mathcal{H}}_t^{(q)}\}} \tag{14}$$

where $o = 1$ if a tweet notifying the corresponding PU was interfered was received, and $o = 0$ otherwise. To simplify (14), notice that $z_{\mathbf{x}}^{(q)}[t] = 1$ implies that $z_{\mathbf{x}'}^{(q)}[t] = 0$ for the grid points $\mathbf{x}' \in \mathcal{G} \backslash \{\mathbf{x}\}$. Then, it follows that $\Pr\{i^{(q)}[t] = 1 | z_{\mathbf{x}}^{(q)}[t] = 1, \tilde{\mathcal{H}}_t^{(q)}\} = \iota_{m,\mathbf{x}}(p_{m,n}^*, \mathbf{s}[t])$. As for the denominator in (14), one only has to average the numerator across all locations. For $i^{(q)}[t] = 1$, this entails

$$\Pr\{i^{(q)}[t] = 1 | \tilde{\mathcal{H}}_t^{(q)}\} = \sum_{\mathbf{x}' \in \mathcal{G}} \iota_{m,\mathbf{x}'}(p_{m^*,n^*}^*, \mathbf{s}[t]) \hat{\beta}_{\mathbf{x}'}^{(q)}[t|t-1]$$

while $\Pr\{i^{(q)}[t] = 0 | \tilde{\mathcal{H}}_t^{(q)}\}$ is computed in the obvious way.

Summarizing, the joint RA and map estimation algorithm amounts to implementing the following steps at each slot t.

[S1] Perform the prediction step (13).

[S2] Based on $\{\beta_{\mathbf{x}}^{(q)}[t|t-1]\}$, find $\{a_m^k[t]\}$, $\{r_{m,n}^k[t]\}$, $\{\bar{p}_m[t]\}$, $\{w_{m,n}[t]\}$, and $\{p_{m,n}[t]\}$ Using Propositions 1-4, where the multipliers are obtained using (12) and the term $\mathbb{E}_{\mathbf{s}[t]}\left[i_{m,n}(p, \mathbf{s}[t])\right]$ in (9) is replaced with $\mathbb{E}_{\mathbf{s}[t]}[i_{m,n}(p, \mathbf{s}[t]) \, | \mathcal{H}_{t-1}] = 1 - \prod_{q=1}^{Q}(1 - \sum_{\mathbf{x} \in \mathcal{G}} \iota_{m,\mathbf{x}}(p) \beta_{\mathbf{x}}^{(q)}[t|t-1])$.

[S3] Acquire the tweets $\{i^{(q)}[t]\}$, and run the correction step (14).

V. Numerical results

To briefly illustrate the merits of the proposed RA approach, consider the scenario depicted in Fig. 1(a), where $M = 12$ SUs are deployed in a 400×400 m area. One flow is simulated, with best-effort traffic (i.e., $a_m^{1,min} = 0$) generated at SUs $\mathcal{N}_S := \{U_1, U_2, U_3, U_4, U_7, U_8\}$, and destined to U_{12}. A PU source communicates with 2 PU receivers using a power of 3 dB. In this setup, the PU receivers are static. The PU system is protected by setting $I = -70$ dB and $i^{\max} = 0.05$.

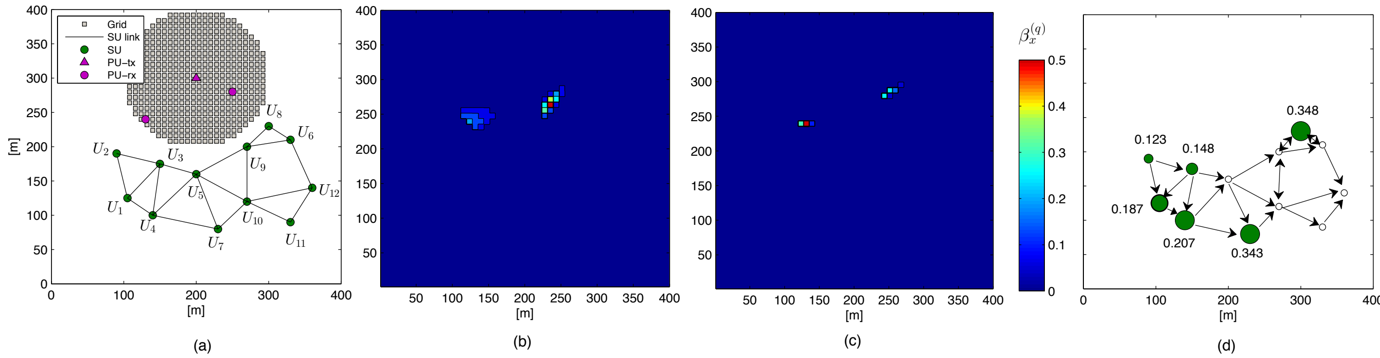

Fig. 1. (a) Scenario. (b) Receiver map at $t = 200$. (c) Map at $t = 2000$. (d) Exogenous rates at $t = 5000$.

The PU coverage region is discretized using uniformly spaced grid points, each one covering an area of 8×8 m. The multipliers are initialized as $\lambda_m^k[0] = 0.1$, $\pi_m[0] = 0.03$, and $\theta[0] = 40$, while the stepsizes are set to $\mu_\lambda = 0.5$, $\mu_\pi = 0.03$, and $\mu_\theta = 0.2$. The proposed scheme is compared with: S1) a scheme where the beliefs are set equal to 1 for grid points on the boundary of the PU coverage region [8]; and, S2) a scheme where perfect PSI (including that of SU-to-PU channels) is available. For S1-S2, $\theta[0]$ is set to 5. Functions $V_m^k = \log_2(a_m^k)$ and $J_m(\bar{p}_m) = \bar{p}_m^2$ are used, and $\kappa_m = 1$ for all U_m. The path loss obeys the model $\gamma_{m,\mathbf{x}} = \|\mathbf{x}_m - \mathbf{x}\|_2^{-3.5}$, while the small-scale fading is Rayleigh distributed.

Pictorially, performance of the receiver localization scheme can be assessed through the maps shown in Figs. 1(b) and (c). The value (color) of a point in the map represents the sum of the beliefs $\sum_q \beta_{\mathbf{x}}^{(q)}[t|t]$ at the corresponding grid point $\mathbf{x}$ [cf. (14)]. The chromatic scale uses blue for low (belief) values and red for high ones. The maps correspond to $t = 100$ and $t = 2,000$, respectively. A uniform distribution across the east and the west halves of the region is used for $\beta_{\mathbf{x}}^{(1)}[0|0]$ and $\beta_{\mathbf{x}}^{(2)}[0|0]$, respectively. It can be seen that after 100 time slots, it is already possible to reveal areas where the PU receivers are likely to reside. Clearly, the localization accuracy improves with time as corroborated by Fig. 1(c).

The average exogenous rates are reported in Fig. 1(d) when $t = 5,000$, along with the links utilized to deliver packets to the destination U_{12}. Since $W = 1$, rates are expressed in bit/s/Hz. Next, convergence and feasibility of the RA schemes are tested in Fig. 2. The overall rate $\bar{a}[t] := (1/t)\sum_m \sum_{\tau=1}^t a_m^1[\tau]$ is depicted in the lower subplot. and it clearly shows that the proposed scheme outperforms S1, motivating the additional complexity required to implement the map estimator. The upper subplot demonstrates that the running average $\bar{i}[t] = (1/t)\sum_{\tau=1}^t i[\tau]$ approaches its limit $i^{\max}$ around $t \approx 4000$. In this case, S1 results in a over-conservative approach. Additional numerical results will be reported in [10].

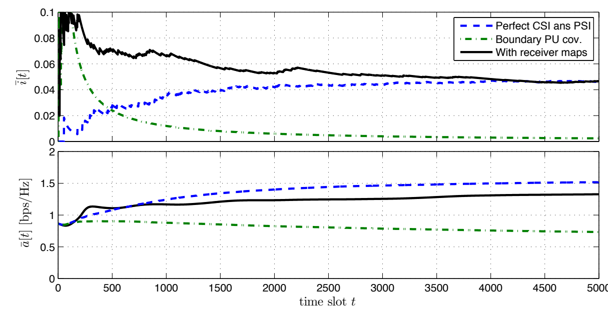

Fig. 2. Convergence and performance of the RA.

REFERENCES

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Sig. Proc. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[2] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[3] Y. Chen, G. Yu, Z. Zhang, H.-H. Chen, and P. Qiu, "On cognitive radio networks with opportunistic power control strategies in fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2752–2761, Jul. 2008.

[4] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[5] S. Huang, X. Liu, and Z. Ding, "Decentralized cognitive radio control based on inference from primary link control information," *IEEE J. Sel. Areas Commun.*, vol. 29, pp. 394–406, Feb. 2011.

[6] A. G. Marques, L. M. Lopez-Ramos, G. B. Giannakis, and J. Ramos, "Resource allocation for interweave and underlay cognitive radios under probability-of-interference constraints," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 10, pp. 1922–1933, Nov. 2012.

[7] K. Eswaran, M. Gastpar, and K. Ramchandran, "Bits through ARQs," 2008, [Online] Available: `http://arxiv.org/pdf/0806.1549.pdf`.

[8] E. Dall'Anese, S.-J. Kim, G. B. Giannakis, and S. Pupolin, "Power control for cognitive radio networks under channel uncertainty," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3541–3551, Dec. 2011.

[9] A. Ribeiro and G. B. Giannakis, "Separation principles in wireless networking," *IEEE Trans. Info. Theory*, vol. 56, no. 9, pp. 4488–4505, Sep. 2010.

[10] A. G. Marques, E. Dall'Anese, and G. B. Giannakis, "Cross-layer optimization and receiver localization for cognitive networks using interference tweets," *IEEE J. Sel. Areas Commun.*, submitted.

[11] A. G. Marques and G. B. Giannakis, "Adaptive cross-layer resource allocation for wireless orthogonal-access networks," in *Proc. of European Wireless Conf.*, Apr 2010.

[12] A. Ribeiro, "Ergodic stochastic optimization algorithms for wireless communication and networking," *IEEE Trans. Sig. Proc.*, vol. 58, no. 12, pp. 6369–6386, Dec. 2010.

[13] Y. Ho and R. Lee, "A Bayesian approach to problems in stochastic estimation and control," *IEEE Trans. Auto. Contr.*, vol. 9, no. 4, pp. 333–339, Oct. 1964.