CHANNEL-ADAPTIVE RESOURCE ALLOCATION FOR COGNITIVE OFDMA RADIOS BASED ON LIMITED-RATE FEEDBACK

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ABSTRACT

Tailored for the emerging class of cognitive radio networks comprising primary and secondary wireless users, the present paper deals with channel-adaptive allocation of subcarriers, rate and power resources for orthogonal frequency-division multiple access (OFDMA). Users rely on adaptive modulation and coding that they select in accordance with the limited-rate feedback they receive from the access point. The access point uses channel state information to maximize the weighted average sum-rate of the network while respecting rate and power constraints on the primary and secondary users. When the channel distribution is available, the optimal off-line allocation is obtained to benchmark performance. In addition, a simple yet optimal on-line algorithm is derived using a stochastic primal-dual approach to solve the constrained utility maximization problem formulated. Analysis and simulations corroborate that the low-complexity online recursive scheme converges to the optimal solution regardless of initialization.

1. INTRODUCTION

In an effort to alleviate today's spectrum scarcity, there has been a growing research interest in cognitive radios, which being aware of their frequency environment can dynamically program radio parameters to efficiently utilize available spectrum without causing harmful interference to authorized users [1]. A major class of cognitive radio platforms typically follows a primary-secondary hierarchy in which primary users hold licences of the spectrum. A secondary user might attempt to coexist with a primary user and access the spectrum *opportunistically*, provided that its presence does not adversely affect the primary spectrum holder(s).

To fulfill the promise of cognitive radios in efficiently utilizing the available bandwidth, it is of paramount importance to judiciously allocate the available channel resources. Since orthogonal frequency division multiple access (OFDMA) is the "workhorse" access technology for most current and proposed wireless standards, the present paper deals with scheduling and resource allocation of OFDMA-based cognitive radios in uplink operation where each (primary or secondary) user transmits to an access point (AP) using channel adaptive modulation and coding (AMC). Specifically, a weighted average sum-rate optimal sub-carrier, power and rate allocation is sought to provide a minimum average rate guarantee for the quality-of-service of the primary user while constraining the maximum rate of secondary users. Scheduling and allocation of resources is adapted to the underlying channel state information (CSI) and is communicated from the AP to all users through a limited-rate feedback channel. The feedback provides quantized CSI for every transmitter (Q-CSIT) to adapt to its intended channel.

For fading channels with known distributions the optimal resource allocation is derived using a primal-dual approach which requires off-line evaluation of the associated optimum Lagrange multipliers (Section 3). This off-line component renders it appropriate for benchmarking purposes and motivates the development of on-line alternatives. Such a simple yet optimal on-line alternative is possible through a stochastic primal-dual (SPD) approach which does not require a priori knowledge of the channel statistics because it learns the required averages on-the-fly (Section 4). The resultant low-complexity, low-overhead on-line resource allocation algorithm is well suited for primary and secondary user access in cognitive radio networks.

2. MODELING PRELIMINARIES

Consider an OFDMA air interface between an AP and $J+1$ wireless users, where user 0 is a primary spectrum holder and users $j=1,\ldots,J$ are secondary users. The overall bandwidth B is divided into K orthogonal narrow-band subcarriers, each with bandwidth B/K small enough to ensure that the fading channel on it is flat, i.e., non-selective. The wireless link between the AP and user j at subcarrier $k=1,\ldots,K$ is then characterized by a random coefficient $\sqrt{h_{j,k}}$. The $(J+1)K\times 1$ vector of channel gains $\mathbf{h}:=\{h_{j,k},\ j=0,1,\ldots,J,\ k=1,\ldots,K\}$ is stationary and ergodic with joint cumulative distribution function (cdf) $F(\mathbf{h})$.

Per subcarrier $k=1,\ldots,K$, we introduce a time-sharing vector $\tau_k(\mathbf{h}):=\{\tau_{j,k}(\mathbf{h}),\ j=0,1,\ldots,J\}$, where entries $\tau_{j,k}(\mathbf{h})$ depend on the channel realization $\mathbf{h}$ and obey the constraint $0\leq\sum_{j=0}^{J}\tau_{j,k}(\mathbf{h})\leq 1$. Specifically, $\tau_{j,k}(\mathbf{h})$ represents the percentage (of time) that user j is allocated (on the average across symbols) access to subcarrier k. If scheduled, i.e., if $\tau_{j,k}(\mathbf{h}) \neq 0$, user j transmits on subcarrier k with a chosen modulation (e.g., 16-QAM) and a channel code (e.g., a convolutional code with rate 1/2) with overall rate $r_{j,m}$. In addition to $\{r_{j,m},\ m = 1,\ldots,M_j\}$ non-zero rates (AMC modes) that can differ from user to user, we let $r_{j,0}$ denote the case where user j does not transmit.

The AP acquires with sufficient training the CSI $\mathbf{h}$ based on which it optimizes resource allocation and feeds back the optimal schedule to the users using a finite number of bits. Such a limited-rate feedback dictates that users have only a limited number of possible transmit-configurations. This implies that for each of the M_j AMC rates, user j can only select power from a set of power values with finite cardinality. It can be shown that if power assignment is optimized based only on Q-CSIT, one must design the optimum quantizer (optimal power-book) jointly with the resource allocation task [2]. This design amounts to having each transmit-mode associated with a unique quantized rate-power pair. Under this design condition, the possible transmit-configurations of user j are the pairs $\{(r_{j,m}, p_{j,m}),\ m = 0,1,\ldots,M_j\}$, where both $r_{j,m}$ and $p_{j,m}$ are constant quantities known to both transmit- and receive-ends. As this paper focuses on resource allocation, it is assumed henceforth that such a power book, i.e, the set of $p_{j,m}$'s is provided.

Reliability of the wireless links must be maintained under a maximum allowable bit error rate (BER) $\check{\varepsilon}$. For constellation- and code-specific constants κ_1 and κ_2 and after assuming without loss of generality (w.l.o.g.) that the additive white Gaussian noise (AWGN) at the receiver has unit variance, the BER with AMC at subcarrier k can be accurately approximated as [3] (our framework applies to any other BER function)

$$\varepsilon_{j,m,k}(h_{j,k}, p_{j,m}, r_{j,m}) = \kappa_1 \exp\left(-\frac{\kappa_2 h_{j,k} p_{j,m}}{2^{r_{j,m}} - 1}\right). \quad (1)$$

Using $\mathbf{h}$, the AP finds for each user j the set of AMC modes satisfying the prescribed BER at subcarrier k, call it $\mathscr{M}_j(h_{j,k}) := \{m : \varepsilon_{j,m,k}(h_{j,k} p_{j,m}, r_{j,m}) \leq \check{\varepsilon}\}$.

It is useful at this point to recognize that except for the pre-specified $r_{j,m}$ $m \in \mathscr{M}_j(h_{j,k})$ modes, each user can also support under the prescribed BER transmit-rates expressed as linear combinations of these AMC modes by time-sharing their usage per subcarrier k. Specifically, using the mode m over $\zeta_{j,m,k}$ percentage of the $\tau_{j,k}$, and letting $\tau_{j,m,k} := \zeta_{j,m,k}\tau_{j,k}$, user j can support rate

$$\tau_{j,k}(\mathbf{h}) r_{j,k}(\mathbf{h}) = \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) r_{j,m} \quad (2)$$

where clearly $0 \leq \sum_{j=0}^{J} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k} \leq 1$. By time-sharing, any linear combination of $\{r_{j,m}\}$ as in (2) gives rise to the same linear combination of corresponding powers $\{p_{j,m}\}$, which meet the pre-specified BER constraint $\check{\varepsilon}$ for a given $h_{j,k}$; hence,

$$\tau_{j,k}(\mathbf{h}) p_{j,k}(\mathbf{h}) = \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) p_{j,m}. \quad (3)$$

Using (2) and (3), it suffices to optimize resource allocation only with respect to the time allocation variables $\tau(\mathbf{h}) := \{\tau_{j,m,k}(\mathbf{h}),\ j = 0,1,\ldots,J,\ m \in \mathscr{M}_j(h_{j,k}),\ k = 1,\ldots,K\}$.

To guarantee quality-of-service of the primary user 0, a minimum average rate $\check{R}_0$ must be maintained for its transmission. On the other hand, to prevent secondary users from abusing the spectrum, maximum average rates $\check{r}_j$ $j = 1,\ldots,J$ should be imposed for these users too. Likewise, average power constraints $\check{P}_0$ and $\check{p}_j$ $j = 1,\ldots,J$ are also present for the primary and secondary users, respectively.

3. WEIGHTED SUM-RATE MAXIMIZATION

With $E_{\mathbf{h}}[\cdot]$ denoting the expectation operator w.r.t. $\mathbf{h}$, the average rate of user $j = 0,1,\ldots,J$ can be is expressed as $\bar{r}_j := E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) r_{j,m}\right]$, and the average power as $\bar{p}_j := E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) p_{j,m}\right]$. Incorporating rate-reward weights $w_j \geq 0$ to effect fairness, the optimal allocation maximizes the weighted average sum-rate subject to (s.to) average rate and power constraints; i.e.,

$$\begin{cases} \max\limits_{\tau(\mathbf{h})} & \sum_{j=0}^{J} w_j E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) r_{j,m}\right] \\ \text{s.to} & C1.\ \ 0 \leq \sum_{j=0}^{J} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) \leq 1;\ \forall \mathbf{h},\ k \\ & C2.\ \ E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_0(h_{0,k})} \tau_{0,m,k}(\mathbf{h}) r_{0,m}\right] \geq \check{R}_0 \\ & C3.\ \ E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_0(h_{0,k})} \tau_{0,m,k}(\mathbf{h}) p_{0,m}\right] \leq \check{P}_0 \\ & C4.\ \ E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) r_{j,m}\right] \leq \check{r}_j,\ j \neq 0 \\ & C5.\ \ E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) p_{j,m}\right] \leq \check{p}_j,\ j \neq 0. \end{cases} \quad (4)$$

The problem formulated as in (4) is convex and can be efficiently solved using a Lagrange multiplier based primal-dual approach [4].

Let λ_{R_0}, λ_{P_0}, λ_{r_j} and λ_{p_j} denote the Lagrange multipliers associated with rate and power constraints of the primary user and the secondary users $j = 1,\ldots,J$, respectively. Ignoring temporarily the trivial constraints C1, the Lagrangian w.r.t. $\lambda := [\lambda_{R_0}, \lambda_{P_0}, \lambda_{r_1}, \lambda_{p_1}, \ldots, \lambda_{r_J}, \lambda_{p_J}]^T$ is

$$\begin{aligned} L(\lambda, \tau) &= \sum_{j=0}^{J} w_j \bar{r}_j + \lambda_{R_0}(\bar{r}_0 - \check{R}_0) - \lambda_{P_0}(\bar{p}_0 - \check{P}_0) \\ &\quad - \sum_{j=1}^{J} \left(\lambda_{r_j}(\bar{r}_j - \check{r}_j) + \lambda_{p_j}(\bar{p}_j - \check{p}_j)\right). \end{aligned} \quad (5)$$

The Lagrange dual function is

$$D(\lambda) = \max_{\tau \text{ s.to C1}} L(\lambda, \tau) \quad (6)$$

and the dual problem of (4) is

$$\min_{\lambda \geq \mathbf{0}} D(\lambda) \quad (7)$$

where $\lambda \geq \mathbf{0}$ means that all entries of λ are nonegative.

Given λ, the optimum in (6) will turn out to be attained almost surely using a greedy winner-takes-all strategy. To establish this, we first define the link quality indicators

$$\varphi_{0,m,k}(\lambda, \mathbf{h}) := (w_0 + \lambda_{R_0}) r_{0,m} - \lambda_{P_0} p_{0,m}, \quad \forall m \in \mathscr{M}_0(h_{0,k}) \quad (8)$$

$$\varphi_{j,m,k}(\lambda, \mathbf{h}) := (w_j - \lambda_{r_j}) r_{j,m} - \lambda_{p_j} p_{j,m}, \quad \forall m \in \mathscr{M}_j(h_{j,k}),\ j \neq 0. \quad (9)$$

Per subcarrier k, we determine for each user $j = 0, 1, \ldots, J$ the most-efficient mode

$$m^*_{j,k}(\lambda, \mathbf{h}) = \arg \max_{m \in \mathscr{M}_j(h_{j,k})} \varphi_{j,m,k}(\lambda, \mathbf{h}); \tag{10}$$

and select the "winner user" as the one with index

$$j^*_k(\lambda, \mathbf{h}) = \arg\max_j \varphi_{j,m^*_{j,k},k}(\lambda, \mathbf{h}). \tag{11}$$

Per sub-carrier k, the optimal schedule of time-sharing fractions is

$$\tau^*_{j,m,k}(\lambda, \mathbf{h}) = \begin{cases} 1, & \text{if } j = j^*_k \text{ and } m = m^*_{j^*_k,k} \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

It is easy to show that the allocation in (12) maximizes the dual function (6). In fact, regarding λ_{P_0} or λ_{p_j} as the power price, the link quality indicators in (8) and (9) determine the net rate reward (rate reward minus power cost) corresponding to the (j,m) mode at subcarrier k. Notice that to guarantee the service of the primary user 0, we may promote its rate reward weight through addition of the multiplier $\lambda_{R_0} > 0$; whereas positive $\lambda_{r_j} > 0$ may be subtracted from w_j to prevent the abusive spectrum access of secondary user $j = 1, \ldots, J$. Using such indicators, the optimal allocation then simply assigns the whole subcarrier k to the winning user-mode pair $(j^*_k, m^*_{j^*_k,k})$ which produces the highest net rate reward. According to such a time allocation, we indeed let user j^*_k exclusively transmit with rate $r^*_{j^*_k,k}(\lambda, \mathbf{h}) = r_{j^*_k, m^*_{j^*_k,k}}$ and power $p^*_{j^*_k,k}(\lambda, \mathbf{h}) = p_{j^*_k, m^*_{j^*_k,k}}$, while having all other users $j \neq j^*_k$ defer by setting $r^*_{j,k}(\lambda, \mathbf{h}) = p^*_{j,k}(\lambda, \mathbf{h}) = 0$ on subcarrier k.

Now we are ready to solve the dual problem (7) for the optimal multipliers λ^*. With $[x]^+ := \max(x, 0)$ and β_i a small stepsize, this can be accomplished through the iterations

$$\lambda^{(i+1)}_{R_0} = \left[\lambda^{(i)}_{R_0} - \beta_i \left(E_{\mathbf{h}}\left[\sum_{k=1}^K r^*_{0,k}(\lambda, \mathbf{h})\right] - \check{R}_0\right)\right]^+ \tag{13}$$

$$\lambda^{(i+1)}_{P_0} = \left[\lambda^{(i)}_{P_0} - \beta_i \left(\check{P}_0 - E_{\mathbf{h}}\left[\sum_{k=1}^K p^*_{0,k}(\lambda, \mathbf{h})\right]\right)\right]^+ \tag{14}$$

$$\lambda^{(i+1)}_{r_j} = \left[\lambda^{(i)}_{r_j} - \beta_i \left(\check{r}_j - E_{\mathbf{h}}\left[\sum_{k=1}^K r^*_{j,k}(\lambda, \mathbf{h})\right]\right)\right]^+ \tag{15}$$

$$\lambda^{(i+1)}_{p_j} = \left[\lambda^{(i)}_{p_j} - \beta_i \left(\check{p}_j - E_{\mathbf{h}}\left[\sum_{k=1}^K p^*_{j,k}(\lambda, \mathbf{h})\right]\right)\right]^+. \tag{16}$$

If the convex problem (4) is strictly feasible, these iterations represent standard sub-gradient projections whose fast (linear) convergence to the optimal λ^* of (7) is guaranteed from any initial non-negative value, e.g., $\lambda^{(0)} = \mathbf{0}$ [5]. The optimal solution for the primal problem (4) is in turn provided by $\tau^*(\mathbf{h}) = \tau^*(\lambda^*, \mathbf{h})$.

It is evident that the expected values in (13)–(16) can be only obtained *off-line* provided that the cdf $F(\mathbf{h})$ is known. Furthermore, notice that the reward weights w_j in (4) are fixed beforehand. On the other hand, practical cognitive radios could welcome *on-line* resource allocation without a priori knowledge of $F(\mathbf{h})$ and with weights adapted to ensure fairness. This becomes possible using stochastic approximation tools and the utility maximization framework (see e.g., [6]) that we outline for our problem in the ensuing section.

4. ON-LINE UTILITY MAXIMIZATION

Select a concave and increasing so called utility function $U_j(\bar{r}_j)$, and consider

$$\begin{cases} \max\limits_{\tau(\mathbf{h})} & \sum_{j=0}^J U_j(\bar{r}_j) \\ \text{s.to} & C1.\ \sum_{j=0}^J \sum_{m \in \mathscr{M}_j(h_{j,k})} \tau_{j,m,k}(\mathbf{h}) \leq 1;\ \forall \mathbf{h},\ k \\ & C2.\ \bar{r}_0 \geq \check{R}_0 \\ & C3.\ \bar{p}_0 \leq \check{P}_0 \\ & C4.\ \bar{r}_j \leq \check{r}_j,\ j \neq 0 \\ & C5.\ \bar{p}_j \leq \check{p}_j,\ j \neq 0 \end{cases} \tag{17}$$

where $\bar{r}_j$ and $\bar{p}$ are defined as in (4). Clearly (17) includes (4) as a special case when $U_j(\bar{r}_j) = w_j \bar{r}_j$. Suppose that the fading coefficients $\mathbf{h}$ remain invariant during a block of OFDMA symbols but can vary from block-to-block (block fading channel model). Aiming at replacing the expectation $\bar{r}_j$, we rely on a standard stochastic approximation on-line recursion across blocks indexed by n block to obtain $\forall j = 0, 1, \ldots, J$

$$\hat{\bar{r}}_j[n+1] = \hat{\bar{r}}_j[n] + \beta_n \left(\sum_{k=1}^K \sum_{m \in \mathscr{M}_j(h_{j,k}[n])} \tau_{j,m,k}(\mathbf{h}[n]) r_{j,m} - \hat{\bar{r}}_j[n]\right); \tag{18}$$

where $\mathbf{h}[n]$ is the fading state at current block n, $\sum_{k=1}^K \sum_{m \in \mathscr{M}_j(h_{j,k}[n])} \tau_{j,m,k}(\mathbf{h}[n]) r_{j,m}$ is clearly the current sum-rate of the user j provided a certain allocation $\tau(\mathbf{h})$, and stepsize $\beta_n \in [0,1]$ implements a forgetting factor in the averaging. Substituting (18) into (17) and using Taylor's expansion with β_n sufficiently small, we have ($'$ denotes derivative)

$$\sum_{j=0}^J U_j(\hat{\bar{r}}_j[n+1]) \approx \sum_{j=0}^J U_j(\hat{\bar{r}}_j[n]) + \sum_{j=0}^J U'_j(\hat{\bar{r}}_j[n]) \times \beta_n \left(\sum_{k=1}^K \sum_{m \in \mathscr{M}_j(h_{j,k}[n])} \tau_{j,m,k}(\mathbf{h}[n]) r_{j,m} - \hat{\bar{r}}_j[n]\right). \tag{19}$$

Since estimates $\hat{\bar{r}}_j[n]$ are available at block n, maximizing $\sum_{j=0}^J U_j(\hat{\bar{r}}_j[n+1])$ becomes equivalent to maximizing the weighted sum-rate $\sum_{j=0}^J U'_j(\hat{\bar{r}}_j[n]) \left(\sum_{k=1}^K \sum_{m \in \mathscr{M}_j(h_{j,k}[n])} \tau_{j,m,k}(\mathbf{h}[n]) r_{j,m}\right)$.

With $\hat{\lambda}[n] := [\hat{\lambda}_{R_0}[n], \hat{\lambda}_{P_0}[n], \hat{\lambda}_{r_1}[n], \hat{\lambda}_{p_1}[n], \ldots, \hat{\lambda}_{r_J}[n], \hat{\lambda}_{p_J}[n]]^T$ denoting the estimated Lagrange multipliers at block n. we can define on-line link quality indicators

$$\varphi_{0,m,k}(\hat{\lambda}[n], \mathbf{h}[n]) := (U'_0(\hat{\bar{r}}_0[n]) + \hat{\lambda}_{R_0}[n]) r_{0,m} - \hat{\lambda}_{P_0}[n] p_{0,m}, \quad \forall m \in \mathscr{M}_0(h_{0,k}[n])$$

$$\varphi_{j,m,k}(\hat{\lambda}[n], \mathbf{h}[n]) := (U'_j(\hat{\bar{r}}_j[n]) - \hat{\lambda}_{r_j}[n]) r_{j,m} - \hat{\lambda}_{p_j}[n] p_{j,m}, \quad \forall m \in \mathscr{M}_j(h_{j,k}[n]),\ j \neq 0.$$

Then arguing as before, we can pick the most efficient user-mode pair $(j^*_k[n], m^*_{j^*_k,k}[n])$ yielding largest $\varphi_{j^*_k, m^*_{j^*_k,k},k}(\lambda[n], \mathbf{h}[n])$, and let user $j^*_k[n]$ transmit with

$r^*_{j^*_k,k}(\hat{\lambda}[n],\mathbf{h}[n]) = r_{j^*_k[n],m^*_{j^*_k,k}[n]}$ and power $p^*_{j^*_k,k}(\hat{\lambda}[n],\mathbf{h}[n]) = p_{j^*_k[n],m^*_{j^*_k,k}[n]}$, while all other users $j \neq j^*_k[n]$ deferring with $r^*_{j,k}(\hat{\lambda}[n],\mathbf{h}[n]) = p^*_{j,k}(\hat{\lambda}[n],\mathbf{h}[n]) = 0$ per subcarrier k.

Substituting this allocation into (18), average rates $\hat{\bar{r}}_j[n+1]$ can be updated. Mimicking the subgadient updates in (13)–(16), we update the $\hat{\lambda}[n+1]$ using the following on-line recursions:

$$\hat{\lambda}_{R_0}[n+1] = \left[\hat{\lambda}_{R_0}[n] - \beta_n\left(\sum_{k=1}^{K} r^*_{0,k}(\hat{\lambda}[n],\mathbf{h}[n]) - \check{R}_0\right)\right]^+$$

$$\hat{\lambda}_{P_0}[n+1] = \left[\hat{\lambda}_{P_0}[n] - \beta_n\left(\check{P}_0 - \sum_{k=1}^{K} p^*_{0,k}(\hat{\lambda}[n],\mathbf{h}[n])\right)\right]^+$$

$$\hat{\lambda}_{r_j}[n+1] = \left[\hat{\lambda}_{r_j}[n] - \beta_n\left(\check{r}_j - \sum_{k=1}^{K} r^*_{j,k}(\hat{\lambda}[n],\mathbf{h}[n])\right)\right]^+$$

$$\hat{\lambda}_{p_j}[n+1] = \left[\hat{\lambda}_{p_j}[n] - \beta_n\left(\check{p}_j - \sum_{k=1}^{K} p^*_{j,k}(\hat{\lambda}[n],\mathbf{h}[n])\right)\right]^+ .$$

Devoid of the expectation operators, these updates are stochastic (un-biased) estimates of the sub-gradient projections in (13)–(16). Such iterations along with the on-line optimal allocation amount to a stochastic primal-dual (SPD) algorithm for solving the utility maximization problem in (17). Per block n, this algorithm performs a weighted sum-rate maximization with adaptive weights provided by $U'_j(\hat{\bar{r}}_j[n])$ to obtain on-line optimal allocation, whereas the dual variables $\hat{\lambda}[n+1]$ are updated using *instantaneous* user rates.

Interestingly, without knowing $F(\mathbf{h})$, this simple SPD on-line algorithm can learn the channel cdf on-the-fly, and is convergent and asymptotically optimal. Specifically, we have proved that:

Proposition 1 *If problem (17) is strictly feasible, then the estimates $\hat{\bar{r}}_j[n]$ obtained recursively from (18) using any initial $\hat{\bar{r}}_j[0] \geq 0$, converge in probability to the optimal $\bar{r}^*_j$ of (17) $\forall j$, as $n \to \infty$ and $\beta_n \downarrow 0$.*

As $\beta_n \downarrow 0$, large samples are used in $\hat{\bar{r}}[n]$ and $\hat{\lambda}[n]$ estimates, which then evolve according to sample averages while satisfying the Lipschitz condition [7]. Ergodicity and stationarity of the channel further implies the equivalence between sample and ensemble averages. Since first-order approximation of the Taylor's expansion used in developing the SPD algorithm also becomes accurate as $\beta_n \downarrow 0$, the evolution of $\hat{\bar{r}}[n]$ maximizes the utility drift; whereas the $\hat{\lambda}[n]$ evolves according to the sub-gradient projection. All of these imply that the trajectory of SPD algorithm indeed fluctuates around that of the corresponding off-line primal-dual updates, and this fluctuation is negligible as $n \to \infty$ and $\beta_n \downarrow 0$. The rigorous proof relying on stochastic approximation and Lyapunov function arguments [7, 8] is omitted due to space limitations.

With a small but constant stepsize $\beta_n = \beta$, the SPD algorithm brings $\hat{\bar{r}}[n]$ to a small neighborhood of $\bar{r}^*$ (with size $o(\beta)$) in $o(1/\beta)$ iterations, uniformly for any initial state. Because this adaptive algorithm converges from arbitrary initializations it exhibits robustness to channel non-stationarities. Upon convergence, the SPD weights become $U'_j(\bar{r}^*_j)$ and the solution of (4) and (17) will coincide if $w_j = U'_j(\bar{r}^*_j)$. Compared to the off-line solution, the adaptive SPD algorithm enjoys two attractive features: i) convergence to the optimal average rates without a priori knowledge of fading cdf, and ii) flexibility in selecting different utility functions to achieve additional desirable properties such as fairness.

5. IMPLEMENTATION AND OVERHEAD

Recall that the AP has available the full CSI vector $\mathbf{h}[n]$ per block n and relies on it to run an SPD iteration. Each iteration includes the on-line optimal allocation as well as the primal $\hat{\bar{r}}[n]$ and dual $\hat{\lambda}[n]$ updates. The scheduled user-mode pairs $\{(j^*_k[n], m^*_{j^*_k,k}[n]),\ k = 1,\ldots,K\}$ are then broadcasted, and users transmit in accordance with this schedule at block n. Instead of the analog-valued vector channel $\mathbf{h}[n]$, the AP needs to feed back to the users the quantized CSI (user and AMC mode indexes selected). Since there are $\sum_{j=1}^{J} M_j$ different user-mode combinations plus one more when all users are deferring, the Q-CSIT (as well as scheduling) overhead is $F = \left\lceil K \log_2\left(\sum_{j=1}^{J} M_j + 1\right)\right\rceil$. As long as the feedback link from the AP to the users can carry more than F bits per block, this Q-CSIT is sufficient for implementing the proposed channel-adaptive resource allocation. Note that this overhead is typically a small number for practical cognitive radios. For instance, in the case of one primary and three secondary users with each supporting $M_j = 5$ AMC modes, only 4 feedback bits are required per subcarrier.

6. NUMERICAL RESULTS

To numerically test our designs, we consider an adaptive OFDMA system with $J = 4$ users, $K = 64$ subcarriers, unitary noise power per user and subcarrier, and $M_j = 5$ active AMC modes per user and subcarrier with rates (1/4,1/2,3/4,1, and 5/4 bits per symbol). We further suppose that subcarrier gains are uncorrelated and constant $\forall k$ (i.e., uncorrelated Rayleigh taps are simulated per user). The utility function is $U_j(\bar{r}_j) = \bar{r}_j$ for all users.

To run our resource allocation algorithms we fixed Q-CSI pairs $\{(r_{j,m}, p_{j,m})\}\ \forall j,m$. Different from this work, [2] deals with optimum rate and power quantization for a TDMA system, obtaining as a result that when optimally designed, the relation between power and rate for different regions is almost constant (i.e., $p_{j,m}/r_{j,m} \simeq c_j\ \forall m$). To facilitate the design of Q-CSI pairs in the present setup, we can use as a rule of thumb the constant c_j based on the BER function in (1) and $\check{\varepsilon}$; since the set of $\{r_{j,m}\}$ rates is given by the AMC modes that users implement, through $\{c_j\}$ we can easily calculate the corresponding $\{p_{j,m}\}$. Specifically, we will obtain c_j as the required power to transmit one bit satisfying $\check{\varepsilon}$ when $h_{j,k} = E_{\mathbf{h}}[h_{j,k}]$.

The QoS constraints are set to: $\check{R}_0 = 30$, $\check{r}_1 = 20$ $\check{r}_2 = 5$, and $\check{r}_3 = 5$ (bits per symbol), $\check{P}_0 = 100$, $\check{p}_1 = 20$ $\check{p}_2 = 100$, and $\check{p}_3 = 20$ (watts per symbol), and $\check{\varepsilon} = 10^{-3}\ \forall j$. The solid line in Figure 1 depicts $\hat{\bar{r}}_j[n]$ and the dashed line $\hat{\bar{p}}_j[n]$ for $\beta_n = 0.05$ and average channel gains across subcarriers and users set to $E_{\mathbf{h}}[h_{j,k}] = 3$. The curves validate the proposed allocation since the requirements are clearly satisfied: the transmit-power is below the maximum value for all users, the primary user transmits at a rate higher than its mini-

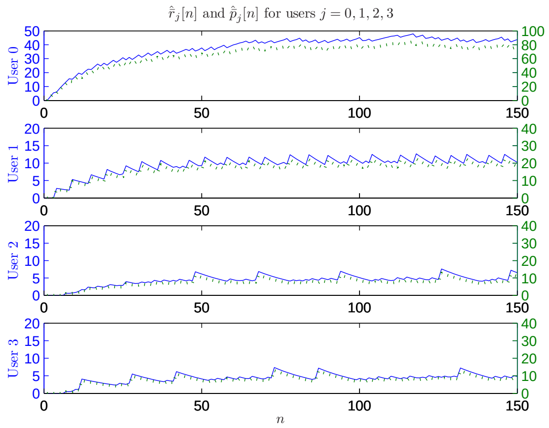

Figure 1: Time evolution of $\bar{\hat{r}}_j[n]$ (solid line, left axis) and $\bar{\hat{p}}_j[n]$ (dashed line, right axis) for $j = 0, 1, 2, 3$.

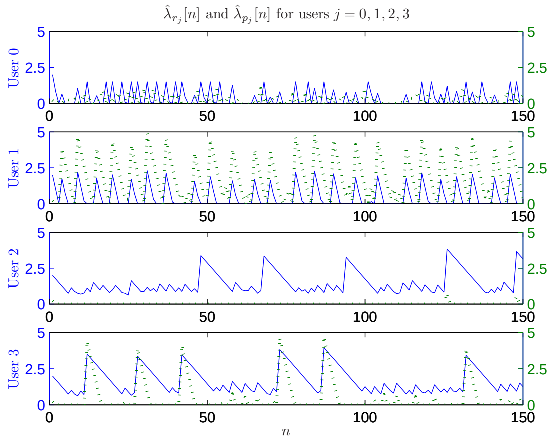

Figure 2: Time evolution of $\hat{\lambda}_{r_j}[n]$ (solid line, left axis) and $\hat{\lambda}_{p_j}[n]$ (dashed line, right axis) for $j = 0, 1, 2, 3$.

mum requirement while the transmit-rates of secondary users stay below their maximum allowable levels. We also observe that to maximize the total transmit-rate $\sum_{j=0}^{J} \bar{r}_j$ the optimal allocation sets: (a) $\bar{r}_j \approx \check{r}_j$ and $\bar{p}_j < \check{p}_j$ for $j = 3, 2$ (in both cases the users have power enough to transmit at higher rate but they would violate their maximum rate constraints); (b) $\bar{r}_1 < \check{r}_1$ and $\bar{p}_1 \approx \check{p}_1$ (although user 1 could transmit at higher rate it does not have enough power); and $\bar{r}_0 > \check{R}_0$ with $\bar{p}_0 < \check{R}_0$ (the primary could slightly increase its own transmit-rate at the expense of reducing the rates of others thus reducing the overall transmit-rate). With respect to convergence, it is fast (but with fluctuations) since we chose a high value for β_n. Moreover, since for $n = 1$ the rate requirement of the primary user is clearly unsatisfied, during the first samples of the algorithm secondary users are not allowed to access subcarriers and their initial rate and power are zero.

To gain insight about the allocation depicted in Figure 1, we plot the corresponding $\hat{\lambda}_{r_j}[n]$ and $\hat{\lambda}_{p_j}[n]$ values in Figure 2. Simply inspection of the latter reveals that: (a) since both rate and power constraints of user $j = 0$ are oversatisfied, its Lagrange multipliers take positive but close to zero values. (Notice that, e.g., $\hat{\lambda}_{r_j}[n]$ is not always zero since it is updated based on the instantaneous values of the transmit-rate and power [cf. (20)], which implies that although $\bar{\hat{r}}_j[n]$ is greater than $\check{r}_j$, $\sum_{k=1}^{K} r_{j,k}$ can be less than $\check{r}_j$ for some n leading the multiplier to take a non-zero value); (b) $\hat{\lambda}_{p_1}[n] > 0$ and $\hat{\lambda}_{r_1}[n]$ is close to zero (i.e., the whole transmit-power is consumed but the maximum transmit-rate is not achieved); (c) both $\hat{\lambda}_{r_2}[n] > 0$ and $\hat{\lambda}_{r_3}[n] > 0$ imply that the maximum rate constraints for $j = 2, 3$ are active; and (d) while $\hat{\lambda}_{p_2}[n] \approx 0$ (in this case the power constraint of user 2 is far from being violated), $\hat{\lambda}_{p_3}[n]$ is close to zero (since $\check{p}_3 \ll \check{p}_2$).

7. CONCLUSIONS

Based on limited-rate feedback, we formulated channel-adaptive resource allocation in cognitive radios with a primary-secondary user hierarchy as a convex optimization problem. Using a Lagrange multiplier based primal-dual apprach, we derived an off-line optimal resource allocation algorithm to maximize the average weighted sum-rate subject to power and rate constraints on the primary and secondary users. We further developed a stochastic approximation SPD algorithm for on-line scheduling and resource allocation. It was argued that this simple low-complexity SPD algorithm asymptotically converges to the optimal off-line resource allocation from any initial value without a priori knowledge of the channel fading statistics.

REFERENCES

[1] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[2] A. G. Marques, X. Wang, and G. B. Giannakis, "Optimizing Energy-Efficient of TDMA with Finite Rate Feedback", *IEEE Proc. of Intl. Conf. on Acoustics, Speech and Signal Processing*, Honolulu, HI, Apr. 16-20, 2007.

[3] A. J. Goldsmith and S. G. Chua, "Adaptive coded modulation for fading channels," *IEEE Transactions on Communications*, vol. 46, no. 5, pp. 595–602, May 1998.

[4] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[5] D. Bertsekas, *Nonlinear Programming*, 2nd Ed., Athena Scientific, 1999.

[6] A. Stolyar, "On the asymptotic optimality of the gradient scheduling algorithm for multi-user throughput allocation," *Operations Research*, vol. 53, no. 1, pp. 12–25, Jan.–Feb. 2005.

[7] H. Kushner and G. Yin, *Stochastic Approximation Algorithms and Applications*, 2nd ed. Berlin, Germany: Springer-Verlag, 2003.

[8] V. Solo and X. Kong, *Adaptive Signal Processing Algorithms: Stability and Performance*, Prentice Hall, 1995.