Underlay Multi-Hop Cognitive Networks with Orthogonal Access

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Abstract

Stochastic algorithms to allocate resources across different layers in an underlay multi-hop cognitive radio with primary and secondary users are presented. The algorithms aim to maximize the utility of the secondary users, while adhering to average interfering power constraints and accounting for the presence of imperfections in the state information. Interference among secondary users is modeled using a binary conflict graph, so that close-by secondary devices cannot transmit simultaneously. The optimal resource allocation dictates the power transmitted by each user, the rates at the transport, network and physical level, and the links to be activated. The design is casted as a nonlinear constrained optimization, and the solution is obtained using stochastic dual decomposition. Numerical experiments validate the theoretical claims.

Keywords

Cognitive radio networks, interference management, dual decomposition, conflict graph.

1. Introduction

Underlay cognitive radios (CRs) can opportunistically use the frequency bands licensed to primary users (PUs), provided that primary communications are not severely disrupted [1], [2]. Hence, the secondary users (SUs) in the CR can take advantage of the spectral opportunities identified through sensing, while keeping the interference inflicted to incumbent users under control [1]–[4]. Non-linear optimization emerges as a natural tool to balance the existing tradeoffs between the primary and secondary systems and to guarantee the required constraints [5], [6]. In the context of underlay CRs, [1] and [2] design optimal power control schemes for single-link CRs under channel uncertainty. Instantaneous and average interference constraints were compared in [3] for the same setup, while an extension to multiple SU links can be found in e.g., [6]–[8].

In this context, the present paper aims to optimally design a multi-hop CR network whose operating conditions are the following. At the network layer, nodes receive packets from different applications, which entail different utility levels [5]. At the link layer, nodes access orthogonally a set of parallel channels. Orthogonal here means that if a terminal is transmitting, no other link interfering with that transmission can be active. The set of links interfering with each other is accounted for using the so-called conflict graph [9]. At the physical layer, nodes can adapt their power and rate loadings in every channel. Finally, *average* flow conservation constraints guaranteeing that the queues are stable [5] and maximum *average* interference constraints limiting the aggregated interference suffered from each of the PUs [8] are considered. The optimization is formulated as a sum-utility maximization that involves variables *averaged* over all possible channel states. The optimal resource allocation (RA) turns out to be a function of the *instantaneous* channel state information (CSI) and the Lagrange multipliers associated with the optimization problem. Since the exact computation of the multipliers is cumbersome, efficient stochastic algorithms that are able to track the optimal values and offer performance guarantees are used [10]. Our algorithms account for the inherent uncertainty present in the state information and are globally optimal. Together with the joint optimization, the main novelty of this work is the consideration of a multi-hop setup with orthogonal access, which leads to modeling SU interference using a conflict graph.[1]

2. Modeling and Problem formulation

Consider a multi-hop SU network comprising I nodes deployed over a given area. The set of nodes that a node i can communicate with constitutes the neighborhood of i and is denoted by $\mathcal{N}_i$. Node connectivity is given by the *undirected* graph $\mathcal{G}$ where

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1. *Notation:* $\mathbb{E}[\cdot]$ denotes expectation; x^* the optimal value of x; $\mathbb{1}_{\{\cdot\}}$ the indicator function ($\mathbb{1}_{\{x\}}=1$ if x is true, and zero otherwise); $[x]_a^b := \min\{\max\{x,a\},b\}$ the projection of the scalar x onto $[a,b]$ and $[x]_+ := [x]_0^\infty$; and $\wedge$ the "and" logical operator.

vertices correspond to SUs and an edge connecting two vertices exists if the corresponding SUs are physically linked. The undirected link between i and j comprises two *directed* links: the i, j link from i to j; and the j, i link from j to i.

We suppose that SUs share a set $\mathcal{F}$ of flat-fading frequency bands indexed by f with an incumbent PU system in an underlay setup [1], [2]. The number of PUs is Q, and they are indexed by q. Not all PUs have to transmit in the same band. The set of carriers used by PU q is denoted by $\mathcal{F}_q$.

The goal of the algorithms in this paper is to use the available CSI (obtained as the output of the sensing phase) to adapt the transmit configuration of the SUs, so that they maximize the SU network performance, while protecting the PU system from *excessive* interference. We assume that PUs *will not* adapt their resources to the needs of the SUs.

2.1. Primary and secondary state information

The system operates in discrete time slots indexed by t. The fth channel's instantaneous power gain from SU i to SU j at time t is denoted by $h_{i,j}^f[t]$; i.e., $h_{i,j}^f[t]$ is the noise-normalized squared magnitude of the fading coefficient [6]. Similarly, the interfering SU-to-PU instantaneous power gain is denoted by $g_{i,q}^f[t]$. The overall channel is described by the CSI vector $\mathbf{s}[t]$ that collects all $h_{i,j}^f[t]$ and $g_{i,q}^f[t]$ gains for instant t. Provided that the spectrum is available for the SUs to transmit, the SU-to-SU channels can be acquired by employing conventional training-based channel estimators. Unless PUs are willing to collaborate (or channel reciprocity can be assumed), acquisition of the SU-to-PU channels is more difficult [1]. For this reason, the imperfections in $g_{i,q}^f[t]$ are expected to be higher than those in $h_{i,j}^f[t]$. When perfect CSI is not available, we will assume that an *instantaneous belief* for all $h_{i,j}^f[t]$ and $g_{i,q}^f[t]$ is available; i.e., the distribution of the current channel value conditioned to all previous measurements. The instantaneous beliefs are denoted by $H_{i,j}^f[t]$ and $G_{i,q}^f[t]$.

2.2. Resources at the secondary network

Transport and network layers: Data packets are generated exogenously by the application layer of each SU, and routed throughout the network to their destination. Packet streams are referred to as flows and indexed by $k = 1, ..., K$. The destination of each flow is denoted by $d(k)$. Different traffic flows (voice, video streaming, elastic data...) may be generated at the same SU. For each flow k, packet arrivals at i are assumed to be random with mean $a_i^k \geq 0$.

Let $r_{i,j}^k[t] \geq 0$ be the *instantaneous*[2] rate used for routing packets of flow k on link i, j. Using the definition in [5], queue stability requires the following condition to be satisfied for all k and $i \neq d(k)$:

$$a_i^k + \sum\nolimits_{j \in \mathcal{N}_i} \mathbb{E}_{\mathbf{s}} \left[r_{j,i}^k(\mathbf{s}) \right] \leq \sum\nolimits_{j \in \mathcal{N}_i} \mathbb{E}_{\mathbf{s}} \left[r_{i,j}^k(\mathbf{s}) \right] \quad (1)$$

SUs implement flow control and adaptive routing, so that $\{a_i^k\}$ and $\{r_{i,j}^k\}$ are variables of the RA problem.

Medium access layer and conflict graph: Let $w_{i,j}^f[t]$ be a binary scheduling variable taking the value 1 if the f-th channel of link i, j is scheduled for transmission during time slot t, and 0 otherwise. SU transmissions are assumed orthogonal. Orthogonal access is adopted by many systems due to its low-complexity. It also enables a (nearly) optimal network operation under moderate-to-strong interference [11]. To find the values of $w_{i,j}^f[t]$ that are feasible, the "so-called" conflict graph $\mathcal{W}$, which is based on the connectivity graph $\mathcal{G}$, has to be built [9]. Vertices in $\mathcal{W}$ are the directed links of $\mathcal{G}$ and an edge connects two vertices in $\mathcal{W}$ if the corresponding directed links cannot be activated simultaneously. In this paper, the conflict graph will be built using the following orthogonality conditions. If link i, j is active, the following links cannot be activated: a) links whose origin or destination is i, b) links whose origin or destination is j, c) links whose destination is a neighbor of i, and d) links whose origin is a neighbor of j. Then, with $\mathcal{E}_{\mathcal{W}}$ denoting the edges in $\mathcal{W}$, the following conditions need to hold for all t

$$w_{i,j}^f[t] + w_{i',j'}^f[t] \leq 1, \text{ if } \big((i,j),(i',j')\big) \in \mathcal{E}_{\mathcal{W}}. \quad (2)$$

An alternative way to write (2) is by using the concept of maximal independent set (MIS), which is a set of vertices such that: i) no two are adjacent, and ii) no other vertex can be added to the set without violating property i). It is then clear that two physical links can be simultaneously activated only if they belong to the same MIS of $\mathcal{W}$. Therefore, finding feasible scheduling policies can be recast as finding MISs of $\mathcal{W}$, which is a difficult problem [9]. To be more concrete, let s be a MIS of $\mathcal{W}$, and let $\mathcal{S}$ be the set containing all the MISs of $\mathcal{W}$. Define then $w_s^f[t] \in \{0, 1\}$ as a binary variable indicating the links in set s that can be activated during slot t. The requirement is now

$$\sum\nolimits_{s \in \mathcal{S}} w_s^f[t] \leq 1, \quad \forall f, t. \quad (3)$$

2. Depending on the context, a generic instantaneous variable x that is adapted based on the instantaneous CSI will be written as $x(\mathbf{s})$, $x(\mathbf{s}[t])$ or $x[t]$.

To take into account the fact that a link can belong to different MISs, let $\mathcal{S}_{i,j}$ denote the set of MISs that contain link (i,j). Then, it needs to hold that

$$w_{i,j}^f[t] \leq \sum\nolimits_{s\in\mathcal{S}_{i,j}} w_s^f[t], \quad \forall f,(i,j). \tag{4}$$

Expressions (3) and (4) can be used to replace (2).

Physical layer: At the physical layer, instantaneous rate and transmit power variables are coupled. For simplicity, here such a rate-power coupling is modeled using Shannon's capacity formula [11]. Hence, if $p_{i,j}^f[t]$ is the power allocated at link i,j, the corresponding rate is $C_{i,j}^f[t] = W\log(1+p_{i,j}^f[t]h_{i,j}^f[t])$, where W is the bandwidth of the channel. Notice that if the CSI is imperfect, the previous has to be replaced with $C_{i,j}^f[t] = \mathbb{E}_{h\in H_{i,j}^f[t]}[W\log(1+p_{i,j}^f[t]h)]$.

Let $\bar{p}_i$ denote the average transmit-power of SU i, which can be expressed as

$$\bar{p}_i = \mathbb{E}_{\mathbf{s}}\Big[\sum\nolimits_{f,j\in\mathcal{N}_i} w_{i,j}^f(\mathbf{s})p_{i,j}^f(\mathbf{s})\Big]. \tag{5}$$

When formulating the optimization, the power transmitted by the SUs will be required to obey two different constraints: $p_{i,j}^f[t] \leq p_i^{f,\max}$ (spectrum mask bounds) and $\bar{p}_i \leq \bar{p}_i^{\max}$ (average power consumption bounds).

2.3.Average interference constraints

To protect PUs from SUs transmissions, constraints limiting the interference must be imposed. Bounds can be enforced on the interfering power, the rate loss or the outage probability [6]–[8]. Moreover, the bounds can be imposed in the short-term (they need to hold for each and every time instant) or in the long-term [3]. The latter are better suited for scenarios with imperfect CSI, but can lead to more frequent service disruption at the PU receivers. Our approach in this paper is to set bounds on the long-term *interfering power* at each PU [8]. Specifically,

$$\mathbb{E}_{\mathbf{s}}\Big[\sum\nolimits_{i,j} w_{i,j}^f(\mathbf{s})p_{i,j}^f(\mathbf{s})g_{i,q}^f\Big] \leq \bar{\iota}_q^{f,max},\ \forall q, f\in\mathcal{F}_q. \tag{6}$$

The reason is twofold: i) the previous constraints will give rise to an optimization problem that is tractable (decomposable across time instants and users), and ii) when the problem is solved using stochastic dual variables, the stochastic iteration can be tuned to render the average constraint close to its instantaneous counterpart. More concrete details on these issues will be provided in the ensuing sections.

3. Problem formulation

The objective of the optimization is designed to penalize high average power consumptions, while promoting high exogenous rates. To this end, let $V_i^k(a_i^k)$ denote a non-decreasing, concave utility function quantifying the reward associated with the exogenous rate a_i^k, and let $J_i(\bar{p}_i)$ be a non-decreasing, convex function representing the cost of SU i using the average power $\bar{p}_i$ [10]. The metric to be maximized is then

$$f(\{a_i^k\},\{\bar{p}_i\}) := \sum\nolimits_{i,k} V_i^k(a_i^k) - \sum\nolimits_i J_i(\bar{p}_i). \tag{7}$$

Hence, with $\mathcal{X} := \{a_i^k,\ \bar{p}_i, r_{i,j}^k(\mathbf{s}), w_{i,j}^f(\mathbf{s}), w_s^f(\mathbf{s}), p_{i,j}(\mathbf{s}), \forall i,j,k,f,\mathbf{s}\}$ collecting all the design variables, the optimal cross-layer RA is the solution to

$$\mathsf{P}^* := \max_{\mathcal{X}} \sum\nolimits_{i,k} V_i^k(a_i^k) - \sum\nolimits_i J_i(\bar{p}_i) \tag{8a}$$

$$\text{subject to}: (1),\ (3),\ (4),\ (6),\ \text{and} \tag{8b}$$

$$\sum\nolimits_k r_{i,j}^k(\mathbf{s}) \leq \sum\nolimits_f w_{i,j}^f(\mathbf{s})C_{i,j}^f(\mathbf{s}, p_{i,j}^f(\mathbf{s})) \tag{8c}$$

$$\mathbb{E}_{\mathbf{s}}\Big[\sum\nolimits_{f,\ j\in\mathcal{N}_i} w_{i,j}^f(\mathbf{s})p_{i,j}^f(\mathbf{s})\Big] \leq \bar{p}_i \tag{8d}$$

$$p_{i,j}^f(\mathbf{s})\in[0,p_i^{f,\max}],\ \bar{p}_i\in[0,\bar{p}_i^{\max}],\ w_s^f(\mathbf{s})\in\{0,1\} \tag{8e}$$

$$w_{i,j}^f(\mathbf{s})\in\{0,1\},\ r_{i,j}^k(\mathbf{s})\geq 0,\ a_i^k\in[a_i^{k,\min}, a_i^{k,\max}] \tag{8f}$$

where $a_i^{k,\min}$ and $a_i^{k,\max}$ are arrival-rate requirements; (8d) has been relaxed and written as an inequality [cf. (5)] without loss of optimality; and (8c) dictates that the rate at the network level cannot exceed the one at the link layer. If needed, (1) and (8c) can be modified to account for losses due to headers.

The presence of binary variables $\{w_{i,j}^f, w_s^f\}$, and the monomials in (6), (8c) and (8d), render the problem non-convex. To deal with the first issue, we implement the relaxation $w_{i,j}^f \in [0,1]$ and $w_s^f \in [0,1]$. After the relaxation, the second issue can be addressed using the properties of the perspective function and the fact that $0 \leq w_{i,j}^f \leq 1$; see, e.g., [10] for details. Hence, the relaxed version of (8) is convex. This does not necessarily imply that it is easy to solve. One of the reasons is that the number of variables and constraints in (8) can be very high. Moreover, one needs to show that the solution of the relaxed problem is feasible for the original one (this issue will be discussed in the next section).

The approach to solve (8) is to dualize the average constraints that couple the resources at different time instants. To this end, let $\{\lambda_i^k\}$, $\{\theta_q\}$, and $\{\pi_i\}$ denote the multipliers associated with (1), (6), and (8d), respectively. After dualization, the Lagrangian is a sum of terms. The approach is to group the terms that depend on the same variables and solve for each of the groups separately; see, e.g., [6] for details. Note that dual decomposition approaches have been successfully applied to many different networking problems [5].

4. Optimal Adaptive RA

Assuming that the optimal multipliers $\mathbf{d}^* := \{\lambda_i^{k*}, \pi_i^*, \theta_q^{f,*}\}$ are available, the optimal primal variables can be computed as follows (proofs are omitted due to space limitations).

Proposition 1. *The optimal $\bar{p}_i^*$ and $\{a_i^{k*}\}$ are found as the solution of the following "scalar" problems*

$$\bar{p}_i^*(\pi_i^*) := \underset{0\leq\bar{p}_i\leq\bar{p}_i^{\max}}{\arg\max}\; - J_i(\bar{p}_i) + \pi_i^*\bar{p}_i\,. \quad (9)$$

$$a_i^{k*}(\lambda_i^{k*}) = \underset{a_i^{k,\min}\leq a\leq a_i^{k,\max}}{\arg\max}\; V_i^k(a) - \lambda_i^{k*}a\,, \quad (10)$$

which are convex. As expected, the optimal flow-control policy (10) takes into account both the reward $V_i^k(a_i^k)$, and the "price" λ_i^{k*} for injecting exogenous traffic at a rate a_i^k into the network. Similarly, the optimal average power in (9) is set by balancing the cost $J_i(\bar{p}_i)$ with the reward represented by π_i^*.

To find the optimal power, scheduling, and routing variables, we need to define first, for each link (i,j), the coefficients [5], [10] $\lambda_{i,j}^{k*} := \lambda_i^{k*} - \lambda_j^{k*}$ and $\lambda_{i,j}^* := \max_k\{\lambda_{i,j}^{k*}\}$, and the functional

$$\varphi_{i,j}^f(\mathbf{s}[t], p, \mathbf{d}^*) := [\lambda_{i,j}^* C_{i,j}^f(\mathbf{s}[t], p) - \pi_i^* p - \sum\nolimits_q \theta_q^{f*}\mathbb{E}_{g\in G_{i,q}^f[t]}[g]p]_+. \quad (11)$$

Using $\lambda_{i,j}^*$ and (11), optimal *instantaneous* powers $\{p_{i,j}^*[t]\}$, scheduling variables $\{w_{i,j}^*[t]\}$, and network rates $\{r_{i,j}^{k*}[t]\}$,are found as specified next.

Proposition 2. *Given $\mathbf{s}[t]$, the optimal $\{p_{i,j}^{f*}[t]\}$ are*

$$p_{i,j}^{f*}[t] := \big[\arg\max_p\; \varphi_{i,j}^{f*}(\mathbf{s}[t], p, \mathbf{d}^*))\big]_0^{p_{i,j}^{f,\max}}. \quad (12)$$

This implies that the optimal nominal power can be found separately for each of the links and channels. Note that each power is aimed to maximize (11), which can be viewed as an *instantaneous* link-quality indicator. The indicator sets a trade-off between the instantaneous transmit-rate, transmit-power and interference, with the multipliers $\lambda_{i,j}^*$, π_i^* and $\theta_q^{f,*}$ representing the corresponding prices. In particular, the third term is used to guarantee that the interference to PUs caused by SU i is kept under control. If the value of $g_{i,q}^f[t]$ is high (strong interference is going to be generated at time t) or if the value of θ_q^{f*} is high (PU q is suffering high interference on channel f), secondary transmissions are discouraged.

Proposition 3. *Given $\mathbf{s}[t]$, the optimal $\{w_{i,j}^{f*}[t]\}$ are*

$$w_{i,j}^{f*}[t] := \sum\nolimits_{s\in S(i,j)} w_s^{f*}[t], \text{ where} \quad (13)$$
$$w_s^{f*}[t] := \mathbb{1}_{\{s=\arg\max_{(s')\in\mathcal{S}}\sum_{(i,j)\in s'}\varphi_{i,j}^f(\mathbf{s}[t],p_{i,j}^{f*}[t],\mathbf{d}^*)\}}.$$

Remarkably, if the maximum in (13) is unique, Prop. 3 states that even after relaxing the binary constraints, the optimal solution is binary. As it will be discussed in the Sec. 5, this is always the case for the stochastic algorithms considered in this paper. Note that Prop. 3 states that only the links belonging to the MIS with maximum aggregated $\varphi_{i,j}^f[t]$ can be active.

Proposition 4. *Per link $(i,j) \in \mathcal{E}$, define the set $\mathcal{K}_{i,j}[t] := \{k : k = \arg\max_j\{\lambda_{i,j}^{j*}\} \wedge \lambda_{i,j}^{k*} \geq 0\}$. Then, the optimal $\{r_{i,j}^{k*}[t]\}$ satisfy the following two conditions: i) if $k \notin \mathcal{K}_{i,j}[t]$, then $r_{i,j}^{k*}[t] = 0$; and, ii) if $|\mathcal{K}_{i,j}[t]| \geq 1$, it follows that $\sum_{k\in\mathcal{K}_{i,j}[t]} r_{i,j}^{k*}[t] = \sum_f w_{i,j}^{f*[t]}\, C_{i,j}^f(\mathbf{s}[t], p_{i,j}^{f*}[t])$.*

Clearly, when $|\mathcal{K}_{i,j}[t]| = 1$, the "winner-takes-all" solution $r_{i,j}^{k*}[t] = \mathbb{1}_{\{k\in\mathcal{K}_{i,j}[t]\}}\sum_f w_{i,j}^{f*[t]}\; C_{i,j}^f(\mathbf{s}[t], p_{i,j}^{f*}[t])$ is optimal. The next section will explain that this is always the case for the algorithms proposed in this paper. Interestingly, it can be shown that if the *constant* optimal $\{\lambda_i^{k*}\}$ are used, the winner flow is not unique and then finding the optimal flow-sharing is more intricate. Prop. 4 establishes that only flows with the highest value of $\lambda_{i,j}^{k*}$ can be routed. The difference $\lambda_{i,j}^{k*} = \lambda_j^{k*} - \lambda_i^{k*}$ can be viewed as a congestion indicator and links to the well-known *back-pressure* routing algorithm can be established [10].

Props. 1 and 4 are similar to those obtained in other wireless networking setups [6]. The main novelties here are the definition of the link-quality indicator in (11) and the results in Prop. 3.

5. Estimating the multipliers

Finding the optimal dual variables $\mathbf{d}^* = \{\lambda_i^{k*}, \theta_q^{f*}, \pi_i^*\}$ may be challenging because classical subgradient methods are computationally expensive and require knowledge of the channel distribution. An effective alternative consists in resorting to stochastic approximation iterations [10], whose goal is to obtain samples $\{\lambda_i^k[t], \theta_q^f[t], \pi_i[t]\}$, $t = 1, 2, \ldots$ that are sufficiently close to the optimal dual variables. The main advantages of using stochastic dual algorithms are: *i)* their computational complexity is much lower than that of their off-line counterparts; *ii)* they can cope with non-stationary environments; and *iii)* they render $\varphi_{i,j}^{f}[t]$ and $\lambda_{i,j}^{k}[t]$ random variables with continuous support, so that the maximizers in Props. 3 and 4 are unique with probability one.

With $\mu_\lambda > 0$, $\mu_\pi > 0$, and $\mu_\theta > 0$ denoting constant stepsizes, the following stochastic dual iterations are proposed:

$$\lambda_i^k[t+1] = [\lambda_i^k[t] + \mu_\lambda(a_i^{k*}(\lambda_i^k[t]) \\ + \textstyle\sum_{j\in\mathcal{N}_i}(r_{j,i}^{k*}[t] - r_{i,j}^{k*}[t])]_+, \quad (14)$$

$$\theta_q^f[t+1] = [\theta_q^f[t] - \mu_\theta(\bar{\iota}_q^{f,\max} \\ - \textstyle\sum_{i,j} w_{i,j}^{f*}[t] p_{i,j}^{f*}[t] \mathbb{E}_{g\in G_{i,q}^f[t]}[g])]_+, \quad (15)$$

$$\pi_i[t+1] = [\pi_i[t] - \mu_\pi(\bar{p}_i^*(\pi_i[t]) \\ - \textstyle\sum_{j\in\mathcal{N}_i} w_{i,j}^*[t] p_{i,j}^*[t])]_+. \quad (16)$$

The update terms in the previous expressions form an *unbiased* stochastic subgradient of the dual function of (8). This feature, together with the fact that the updates are bounded, can be used to show that the stochastic RA is asymptotically feasible; i.e., that as $t\to\infty$ it holds that $\frac{1}{t}\sum_{\tau=1}^{t}\sum_{i,j} w_{i,j}^{f*}[t] p_{i,j}^{f*}[t] g_{i,q}^{f}[t] \le \bar{\iota}_q^f$ for all (q,f). Moreover, it can also be shown that the loss of optimality relative to P^* [cf. (8a)] is proportional to μ; hence, it vanishes if $\mu\to 0$. Details are not presented due to space limitations, but the proof relies on the convergence of stochastic subgradient methods [10].

Signaling and computational overhead: The main challenges to implement the proposed schemes are: i) the update of $\theta_q^f[t]$, which requires either exchange of information among the SUs that are interfering the specific PU, or collaboration of the PU receiver; and ii) the computation of $w_{i,j}^{f*}[t]$ according to Prop. 3. Regarding ii), the number of MISs can be very high, so that finding all of them can entail a high cost (at least, this computation has to be run only once and not for every t). Moreover, to identify the MIS with maximum aggregated $\sum_{(i,j)\in s}\varphi_{i,j}^{f*}[t]$, the SUs must broadcast their quality link indicators $\varphi_{i,j}^{f*}[t]$. Errors in these tasks will be considered in Sec. 6, but rigorous design of robust schemes is left as a future work.

6. Numerical results

Consider the heterogeneous network in the right panel of Fig. 1 (for now, ignore the values of a_i^k shown in the figure), with $I=25$ SU nodes and $Q=2$ PUs (marked with red ellipsoids). The SUs interfering with the PUs are marked with green circles. Node $i=1$ is a Macro Base Station (M-BS) and nodes $2\le i\le 7$ are micro BSs. BSs $i=2,3,4$ are linked to the M-BS via a wireless link and BSs $i=5,6,7$ are linked via a wire channel (free of interference). All other channels are Rayleigh-distributed with a path loss exponent of 3.5 [11]. Three different flows are simulated: a) traffic from all terminal nodes ($i\ge 8$) to node $i=1$; b) traffic from $i=10$ to $i=17$; and c) traffic from $i=16$ to $i=17$ (same cell). PUs are protected by setting $\bar{\iota}_q^{f,\max}=0.01$. The remaining parameters are set to $V_i^k(x)=\log(0.1+x)$, $J_i(x)=x^2$, $a_8^{1,\min}=6$ and $W=1$. Since our schemes are globally optimal, the main goal of the simulations is to gain insights on their behavior. Additional results and setup details will be given in the journal version of this paper.

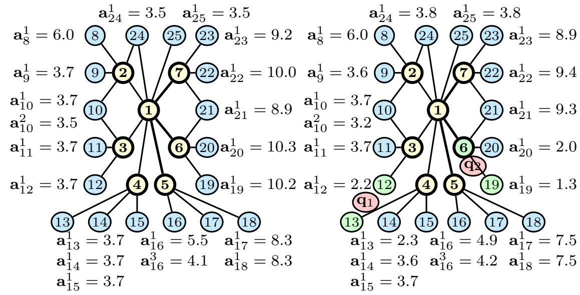

Figure 1. Network w/ $Q=0$ (left) and $Q=2$ (right).

The first test case (TC1) evaluates the performance for 3 different algorithms: TC1-1) the schemes in this paper, but ignoring the interference to the PUs; TC1-2) the schemes in this paper; and TC1-3) the optimal solution replacing the average interference constraints with instantaneous ones. The global objectives, sum-rate and sum-power are: 31.6, 117, 2.8 (TC1-1); 26.8, 94, 1.8 (TC1-2); and 24.1, 85, 2.5 (TC1-3). The values for $\{a_i^k\}$ for TC1-1 and TC1-2 are shown in Fig. 1. The values of a_i^k for the SUs interfering the PUs in TC1-3 are: $a_{12}^1=1.7$, $a_{13}^1=1.7$ and $a_{19}^1=1.9$. The first observation is that all the solutions obey their corresponding constraints. Moreover, TC1-1 achieves the best performance (it is the least constrained of the three problems) and TC1-2 achieves a better objective than TC1-3. The latter is reasonable because the solution to TC1-3 is feasible for TC1-2, but the opposite is not true.

TC2 assesses the feasibility and convergence of the long-term interference constraint. Fig. 2 shows the average interference as a function of time. Defining $\iota_q^f[t] := \sum_{i,j} w_{i,j}^{f*}[\tau] p_{i,j}^{f*}[\tau] g_{i,q}^f[\tau]$, the three curves in Fig. 2 represent: TC2-1) the running average of the interference $\bar{\iota}[t] := (1/t)\sum_{\tau=1}^{t}\iota_q^f[\tau]$ of scheme TC1-2 (the one proposed in the paper); TC2-2) the

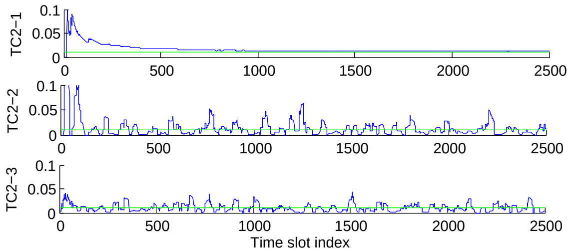

Figure 2. Average interference rate.

finite length windowed average interference $\bar{\iota}_L[t] := (1/L)\sum_{\tau=t-L+1}^{t} \iota_q^f[\tau]$ of scheme TC1-2 with $L = 25$; and TC2-3) the counterpart of TC2-2 when the RA and the multipliers are updated 5 times for every t. The main observation is that the average interference is satisfied after a few hundreds of slots (cf. TC2-1). The results for TC2-2 confirm that the constraint is not satisfied in the medium term. In fact, $\bar{\iota}_L[t] > 3\iota^{\max}$ during, approximately, 10% of the time. More interestingly from a practical perspective, the results for TC2-3 demonstrate that updating the RA and the multipliers at a faster rate is an effective way to reduce the constraint violation in the medium term (for TC2-3, $\bar{\iota}_L[t] > 3\iota^{\max}$ only during 2% of the time).

TC3 investigates the RA in the presence of errors, namely: TC3-1) errors when estimating the SU-to-PU channels $g_{i,q}^f[t]$ and TC3-2) errors when reporting the values of $\varphi_{i,j}^{f*}[t]$ (this tries to emulate scenarios where the scheduling is solved approximately, so that the optimal set of links is not always activated). The optimality loss with respect to the solution in TC1-2 based on perfect CSI are: 3% (TC3-1 with a 10% error); 5% (TC3-1 with a 20% error); 20% (TC3-1 with only statistical CSI); 2% (TC3-2 with a 10% error); and 7% (TC3-2 with a 20% error). Although for TC3-1 the loss is small, if we focus solely on the SUs that interfere the PUs, the loss is close to 60%. In all cases the interference constraints remain feasible. Hence, the results show that, for the tested scenarios, the schemes are robust to errors and, as expected, that the optimality loss is higher as the errors grow larger.

7. Concluding remarks

Adaptive cross-layer RA schemes for multi-hop CRs were designed using stochastic dual decomposition techniques. SU transmissions obey orthogonality constraints captured by a conflict graph and take into account the long-term interference generated to the PUs. Algorithms able to find the globally optimum solution were developed. The main implementation challenges were associated with the computation of the optimal link scheduling and the interference multipliers. Preliminary numerical results illustrated the theoretical claims and showed robustness to imperfections in the CSI.

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