POWER-EFFICIENT OFDM WITH REDUCED COMPLEXITY AND FEEDBACK OVERHEAD*

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ABSTRACT

Motivated by the increasing demand for low-cost low-power wireless sensor networks and related applications, we develop suboptimal but simple bit and power loading algorithms that minimize transmit-power for orthogonal frequency division multiplexing (OFDM) under rate and error probability constraints. Bit and power loading adaptation are based on a quantized version of channel state information (D-CSI) conveyed from the receiver to the transmitter. Our design exploits the correlation among sub-carriers in order to reduce feedback overhead. Numerical examples support our claim that simple suboptimal schemes with a reduced number of feedback bits achieve near-optimal performance while providing significant power savings.

1. INTRODUCTION

A great deal of research has been recently devoted to communication systems operating under stringent power constraints, wireless sensor networks offering a popular example. For use by these systems, we derive in this paper an easy-to-implement power-efficient orthogonal frequency division multiplexing (OFDM) scheme. For its spectral efficiency, error resilience, and ease of implementation, OFDM has been adopted by digital subscriber lines (DSL), digital audio and video broadcasting (DAB/DVB) standards and wireless local area networks, to name a few [6, 11]. OFDM transmissions over wireline or slowly fading wireless links have traditionally relied on deterministic (per channel realization) channel state information at transmitter (CSIT) to adaptively load power and bits so as to maximize rate (capacity) for a prescribed transmit-power constraint. However, errors in estimating the channel at the receiver, feedback delay, and the asymmetry between forward and reverse links render acquisition of deterministically perfect CSIT pragmatically impossible in most wireless scenarios. This has motivated OFDM loading schemes based on statistical (S-) or quantized (Q-) CSIT, whereby a limited number of bits are fed back from the transmitter to the receiver; see e.g., [2, 8, 12] and references therein. But these works too deal with the bandwidth-limited setup, where the objective is to maximize rate or minimize bit error rate (BER).

Interestingly, except for [10] where deterministic (D-) CSIT is used to minimize power consumption, analogous efforts have not been devoted to optimizing OFDM in the *power-limited* regime. Under prescribed transmission rate and bit error rate (BER) constraints, we have recently devised OFDM schemes consuming minimum power based on Q-CSIT [9]. Our focus on Q-CSIT is well justified since the resultant transceivers are universally applicable to frequency-selective wireless channels, they incur controllable amount of feedback overhead and implementation complexity, they turn out to be more power-efficient than S-CSIT based ones, and for a sufficient number of feedback bits they even approach the power savings achieved by the benchmark D-CSIT designs. To facilitate implementation of this Q-CSIT based design, our goal in this paper is to simplify the associated optimization tasks and reduce the required feedback overhead.

The rest of the paper is organized as follows. Section 2 describes the setup. Reduced complexity alternatives are studied in Section 3. Means of reducing feedback overhead is offered in Section 4. Finally, numerical results and conclusions are provided in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

We consider a point-to-point wireless OFDM transmission using N sub-carriers (sub-channels) through a frequency-selective channel with discrete-time baseband equivalent impulse response taps $\{h_n\}_{n=0}^{L}$, where: $L := \lfloor D_{\max} W \rfloor$ denotes the channel order, $D_{\max}$ its maximum delay spread, W the transmit-bandwidth, and $\lfloor\cdot\rfloor$ stands for the floor operation. The data stream of each sub-channel, indexed by $k \in [0, N-1]$, is loaded with a constant power P_k and modulated to yield symbols drawn from a constellation of size M_k. The application of N-point inverse fast Fourier transform (I-FFT) to the N symbol streams followed by insertion of a size-L cyclic prefix (CP) converts the multipath fading frequency-selective channel to a set of N parallel flat-fading sub-channels upon performing the reverse operations at the receiver side. The fading coefficient of each sub-channel is given by the FFT: $H_k = (1/\sqrt{N}) \sum_{n=0}^{N-1} h_n e^{-j\frac{2\pi}{N}kn}$, where N is typically chosen so that $N \gg L$.

With the channel acquired (e.g., via pilot symbols), the receiver has available a noise-normalized channel gain vector $\mathbf{g} := [g_0, \ldots, g_{N-1}]^T$, where $[.]^T$ denotes transposition and $g_k := |H_k|^2/\sigma_k^2$ is the instantaneous noise-normalized channel gain of the kth sub-channel on which the zero-mean additive white Gaussian noise (AWGN) has variance σ_k^2. While each deterministic realization of the random gain g_k constitutes the deterministic CSI (D-CSI), the region index where g_k belongs represents Q-CSI. Specifying the form

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of Q-CSI entails selection of quantization thresholds $\tau_{k,j}$, which in turn define quantization regions $\mathcal{R}_{k,j}$ each sub-channel realization g_k falls into. The random vector of bits $\mathbf{j} := [j_0, j_1, \ldots, j_{N-1}]^T$ representing indices of the regions of all sub-channels is what we term Q-CSI.

The underlying system assumptions are:

a1. *Symbols are drawn from quadrature amplitude modulation (QAM) constellations of size* M_k*;*

a2. *Sub-channels* H_k *remain invariant over at least two consecutive OFDM symbols, and each is complex Gaussian distributed; i.e., each sub-channel gain* g_k *adheres to an exponential probability density function (pdf)* $f_{g_k}(g_k) = (1/\bar{g}_k)\exp(-g_k/\bar{g}_k)$*;* and

a3. *Feedback is error-free and incurs negligible delay.*

Assumptions a1-a3 are common to existing designs [6, **?**, 2, 8], and are typically satisfied in practice. In principle, our results apply to any channel pdf, but a2 simplifies the resultant designs. Channel invariance in a2 allows for feedback delays; it may be stringent for D-CSIT based designs, but is very reasonable in the Q-CSIT case since each sub-channel can vary from one OFDM symbol to the next so long as the quantization region it falls into remains invariant. Finally, error-free feedback is easily guaranteed with sufficiently strong error control codes especially since rate in the reverse link is low.

With R denoting the number of bits per OFDM symbol (rate) and P_k denoting the power per sub-channel, the instantaneous BER is

$$\mathrm{BER} = \frac{1}{R}\sum_{k=0}^{N-1}\log_2(M_k)\mathrm{BER}_k(g_k, P_k, M_k), \quad (1)$$

where for the QAM constellations in a1, we have as a tight approximation [4, 1]: $\mathrm{BER}_k(g_k, P_k, M_k) \simeq a\exp(-\beta_k g_k P_k)$ with $a = 0.2$, $\beta_k := \frac{b_k}{M_k - 1}$, and $b_k = 1$ if $M_k = 2$, or, $b_k = 1.5$ if $M_k \geq 4$.

System design comprises two phases: (i) *off-line* phase that selects a proper channel quantizer and (ii) *on-line* phase that capitalizes on the channel gains specified by the bit vector $\mathbf{j}$ (Q-CSIT) to choose power and bit vectors $\mathbf{p} := [P_0 \ldots P_{N-1}]^T$ and $\mathbf{m} := [M_0 \ldots M_{N-1}]^T$ for loading across sub-carriers. The two phases aim at minimizing the power $P := \sum_{k=0}^{N-1} P_k$, or its average $\bar{P}$, for prescribed values of the rate (R_0) and the bit error rate (BER_0) per OFDM symbol.

Adopting the average total power as a quantization distortion metric, the off-line problem can be formulated as:

$$\begin{aligned}&\min_{\tau_{k,j}\geq 0} J(\bar{\mathbf{p}}),\ \ J(\bar{\mathbf{p}}) := \textstyle\sum_{k=0}^{N-1}\bar{P}_k\\ &\text{s. to}\\ &\text{C1. } \textstyle\sum_{k=0}^{N-1}\frac{\log_2 M_k}{R_0}\overline{\mathrm{BER}}_k(\tau_k, \{P_{k,j}\}, M_k) = \mathrm{BER}_0\\ &\text{C2. } \textstyle\sum_{k=0}^{N-1}\log_2 M_k = R_0\ ,\end{aligned} \quad (2)$$

where $\tau_k := [\tau_{k,1}, \cdots, \tau_{k,2^B-1}]^T$. One should notice that optimization with respect to $\tau_{k,j}$ involves optimization over M_k and $\bar{P}_k$ as well. This in turn yields as a byproduct $(\mathbf{m}, \mathbf{p})$ loadings that can be used constantly in the on-line operation as a means of reducing on-line complexity; see also Section 3.2.

Now given a quantizer design that results from (2), the receiver can easily construct a codeword $\mathbf{j}$ (Q-CSI) which is fed back to the transmitter. This will yield during the on-line phase the loadings $(\mathbf{m}, \mathbf{p})$ by solving the optimization problem

$$\begin{aligned}&\min_{\mathbf{m}\in\mathcal{M}, \mathbf{p}\geq \mathbf{0}} J(\mathbf{p}),\ J(\mathbf{p}) := \textstyle\sum_{k=0}^{N-1} P_k\\ &\text{s. to}\\ &\text{C1.}\textstyle\sum_{k=0}^{N-1}\frac{\log_2 M_k}{R_0}\overline{\mathrm{BER}}_{k|\mathbf{j}}(\tau_{k,j}, \tau_{k,j+1}, P_k, M_k|\mathbf{j}) = \mathrm{BER}_0\\ &\text{C2.}\textstyle\sum_{k=0}^{N-1}\log_2 M_k = R_0\end{aligned} \quad (3)$$

where $\mathcal{M}$ is the finite set of possible values of $\mathbf{m}$ and

$$\overline{\mathrm{BER}}_{k|\mathbf{j}} := \frac{\int_{\tau_{k,j}}^{\tau_{k,j+1}}\mathrm{BER}_k(P_k, g_k) f_{g_k}(g_k)dg_k}{\Pr(g_k \in \mathcal{R}_{k,j})} \quad (4)$$

is what we term conditional average BER (given regions indices).

Optimum, albeit complex, algorithms to solve (2) and (3) were given in [9] using the Karush-Kuhn-Tucker (KKT) conditions and the Lagrangian technique [5, Sec. 14.6]. These algorithms will be referred to as Q-CSIT solutions. We next focus on suboptimal but simple to obtain solutions.

3. REDUCED COMPLEXITY OPTIMIZATION

Due to the integer nature of $\mathbf{m}$, the optimization tasks listed of the previous section can be carried out repeatedly, over $\bar{\mathbf{p}}$ and $\tau_{k,j}$ for the off-line phase or just over $\mathbf{p}$ for the on-line phase, for each feasible $\mathbf{m}$ that achieves R_0. Eventually, the best candidate will be chosen. In turn, given B bits to quantize each sub-channel (with 2^B regions), per iteration of the quantizer design (corresponding to each feasible $\mathbf{m}$), we look for $N(2^B-1)$ thresholds $\tau_{k,j}$, and $N(2^B-1)$ power levels $P_{k,j}$ which requires searching over $2N(2^B-1)$ unknowns. Likewise, the on-line search space entails $N(2^B-1)$ unknowns to solve for $P_{k,j}$ in all sub-channels for each $\mathbf{m}$. As N and B increase, this motivates well the reduced complexity, albeit suboptimal, schemes we develop in this section. The key step in reducing complexity eliminating half of the unknowns in both phases of the optimization process by expressing the power levels $P_{k,j}$ in terms of the thresholds $\tau_{k,j}$. Our approach then is to start from the D-CSIT problem (similar to (3) while replacing $\overline{\mathrm{BER}}_{k|\mathbf{j}}(\tau_{k,j}, \tau_{k,j+1}, P_k, M_k)$ by $\mathrm{BER}_k(g_k, P_k, M_k)$) to obtain an optimum power loading $P_k(g_k)$ that turned out to be [9]

$$P_k(g_k) = \frac{1}{\beta_k g_k}\left[\ln\left(\frac{a\beta_k g_k\log_2(M_k)}{R_0}\lambda_1\right)\right]^+, \quad (5)$$

where $[x]^+ := \max(x, 0)$, and λ_1 will be termed in this context as a power quantization parameter that has to be solved jointly with the Lagrange multiplier λ_Q corresponding to the Q-CSIT optimization problems in (2) and (3). Different reduced complexity (RC) alternatives can then use (5) to express $P_{k,j}$ as a function of $\tau_{k,j}$:

RC1: $P_{k,j} = P_k(\hat{g}_k) = P_k(\frac{\tau_{k,j}+\tau_{k,j+1}}{2})$;

RC2: $P_{k,j} = P_k(\hat{g}_k) = P_k(\mathbb{E}[g_k|\tau_{k,j} < g_k < \tau_{k,j+1}])$;

RC3: $P_{k,j} = \frac{1}{2}P_k(g_k = \tau_{k,j}) + \frac{1}{2}P_k(g_k = \tau_{k,j+1})$;

RC4: $P_{k,j} = \mathbb{E}[P_k(g_k)|\tau_{k,j} < g_k < \tau_{k,j+1}]$.

Among these alternatives, coupled with the simulation results, the RC3 alternative (power middle point) is the most promising to calculate $P_{k,j}$ when considering both complexity and power efficiency. Eventually, the search space in the on-line phase reduces to searching only over λ_1 and the Lagrange multiplier λ_Q when solving the optimization problem using the KKT conditions; whereas in the off-line phase, the search space comprises the two λ's as well as $N(2^B - 1)$ unknown thresholds.

3.1. Reduced Complexity: Off-Line Phase

In this section, we seek further complexity reduction in the off-line phase by considering two suboptimal schemes.

3.1.1. Common Thresholds (CT) for all Sub-channels

Here we drop the threshold subscript k and use $\tau_{k,j} = \tau_j$ for all k, which leads to an N-fold reduction in the threshold search space (only $2^B - 1$ unknowns are involved for each choice of $\mathbf{m}$) and the resulting common thresholds (CT) scheme shall be referred to as CT-QCSIT. Straightforward application of the KKT conditions to (2) yields the optimum $2^B - 1$ common thresholds [9]. Upon solving for the thresholds and λ_1, $P_{k,j}$ can be obtained as explained earlier using RC3 and (5).

3.1.2. Sub-carrier Irrespective Thresholds (SIT)

When optimally solving (2), i.e., without any simplifications, we observed as a rule of thumb that for each j, we have $\tau_{k,j} \simeq c_j \bar{g}_k$ for some constant c_j irrespective of k. This motivates us to look for designs that quantize sub-channels individually. Notice that unlike CT, this SIT based design accounts for the individual sub-channel means, and depending on the distortion metric adopted we can devise two simplified quantizers: one enforcing equi-probable (EP) quantization regions (EP-QCSIT scheme), and another minimizing the mean-square error in quantizing either the gains (what we naturally term Lloyd-g_k quantizer), or, the powers (what we call Lloyd-P_k quantizer) [7]. For EP Quantization, we determine $\tau_{k,j}$'s so that $\Pr(g_k \in \mathcal{R}_{k,j}) = \int_{\tau_{k,j}}^{\tau_{k,j+1}} f_{g_k}(g_k) dg_k = 1/2^B \; \forall k, j$. Under the pdf in a2, this yields $\tau_{k,j} = \bar{g}_k \ln \frac{2^B}{2^B - j}$ which interestingly adheres to the aforementioned rule of thumb and as such asserts a means of optimality to the EP quantizer.

For Lloyd Scalar Quantization, we adopt the scalar Lloyd algorithm [7, 3], which for a random variable x with pdf $f_x(x)$ quantized to $\hat{x}$ (using 2^B levels) is known to minimize $\mathbb{E}[(x - \hat{x})^2]$. The quantized $\hat{x}$ and thresholds τ_j, are obtained by iteratively solving in an alternating fashion the equations: $\tau_j = \frac{1}{2}(\hat{x}_j + \hat{x}_{j+1})$ and $\hat{x}_j = \frac{\int_{\tau_j}^{\tau_{j+1}} x f_x(x) dx}{\int_{\tau_j}^{\tau_{j+1}} f_x(x) dx}$. We can apply Lloyd's algorithm to our problem with either $x = g_k$ or $x = P_k$ (what we referred to as Lloyd-g_k or Lloyd-P_k quantizers).

3.2. Reduced Complexity: On-Line Phase

As illustrated before, the search space is now confined to finding λ_1 and λ_Q for each feasible $\mathbf{m}$ entry in the on-line phase. Further reduction in complexity can be accomplished by utilizing fixed parameters that resulted from the off-line phase. In turn, the transmitter can operate in any of the following modes (sorted in decreasing power efficiency but also in decreasing complexity order):

Tx A: select optimally the set of active sub-carriers $\mathcal{N}_a$, bit loading $\mathbf{m}$, and the values of λ's, to attain the global optimum $\mathbf{p}$;
Tx B: select optimally $\mathcal{N}_a$, λ's while using a fixed $\mathbf{m}$;
Tx C: select optimally $\mathcal{N}_a$, but use fixed values for $\mathbf{m}$, and λ's; or
Tx D: select optimally λ's, but use always the same $\mathcal{N}_a$ and $\mathbf{m}$.

It is then up to the designer to select among TxA-TxD depending on application specific constraints.

4. REDUCED FEEDBACK OVERHEAD

In this section, we seek reduction in the NB required feedback bits. To this end, we will exploit the statistical dependence among sub-channels which prevalent when $N \gg L$. It is not difficult to prove that [9]

Proposition 1: *Only $(2L+1)B$ bits suffice to quantize the channel gain vector* $\mathbf{g}$.

For $N \gg L$, Proposition 1 reduces the required number of feedback bits considerably: from NB to $(2L+1)B$. Because $2L+1$ samples of g_k on the FFT grid suffice to fully identify the entire gain profile, we let S denote the set of the indices of the $2L+1$ uniformly sampled sub-channels and S_c its complement. Given j_k, we first form the estimates $\hat{g}_k$ for $k \in S$ and then interpolate to obtain the remaining g_k's for $k \in S_c$. Notice that any of the off-line schemes described in the preceding section can be used first to solve for the thresholds in all sub-channels. Then, the estimation/interpolation task is executed at the transmitter during the on-line phase. Afterwards, any of the on-online algorithms discussed in the previous section can be implemented (TxA-TxD). Figure 1 depicts the different computational modules along with their interactions. The first task of the estimation/interpolation module, namely sub-channel estimation, can be accomplished by using any of the four RC options discussed in the previous section. In RC3 and RC4, we estimate power and then use (5) to estimate g_k for $k \in S$. Given $\hat{g}_k$ for $k \in S$, our next task is to interpolate to obtain g_k for $k \in S_c$. Various interpolators are possible but our numerical results suggest that the *sinc* interpolator leads to the most power efficient scheme.

5. NUMERICAL EXAMPLES AND CONCLUSIONS

To numerically test our power-efficient designs, we consider $N = 64$ sub-carriers, $B = 2$ bits, $\{\bar{g}_k\}_{k=0}^{N-1}$ varying over a 10dB range, $\text{BER}_0 = 10^{-3}$, and $R_0 = 20$ bits. We compare our simple suboptimal schemes to the optimum Q-CSIT and D-CSIT ones [9]. In addition to its simplicity, EP-QCSIT was numerically found to yield the best performance among the different SIT reduced complexity schemes. For this reason, it will be chosen to represent this class.

Test Case 1 (D-CSIT vs. Q-CSIT Schemes): In Figure 2, three schemes utilizing Q-CSIT are tested (optimum Q-CSIT, CT-QCSIT, and EP-QCSIT) for the different adaptive transmitters TxA-D. We observe that all four strategies TxA-D perform within 2 dB with respect to each other. In addition, the suboptimal EP-QCSIT scheme shows near-optimum power efficiency approximately 0.2 dB away from the optimal Q-CSIT and almost 1 dB away from the optimal D-CSIT benchmark with fully adaptive transmission (Tx-A).

Test Case 2 (Reduced Complexity Optimization and Feedback Overhead): Here we test sub-optimal schemes with controllable feedback overhead which rely on RC1-RC4 sub-channel estimation options as well as sinc interpolation. Figure 3 depicts the power efficiency of the various estimation options in comparison with D-CSIT and Q-CSIT without reduced overhead feedback. Thanks to its simplicity and accuracy, RC2 used as amplitude estimation method is preferable if the region is given. Notice also the negligible 0.3dB loss in average total power relative to feedback-per-subchannel Q-CSIT, and the minimal 1 dB loss compared to the D-CSIT scheme.

In conclusion, we have proposed a power efficient OFDM scheme that is practically appealing in the sense that it relies on quantized feedback information and exploits the correlation among sub-channels. We have further devised different alternatives for reducing both computational complexity and feedback overhead. Interestingly, compared to the deterministically and statistically based designs, the proposed quantized based design exhibits, respectively, about 2 dB loss and 15 dB gain from the power efficiency perspective.

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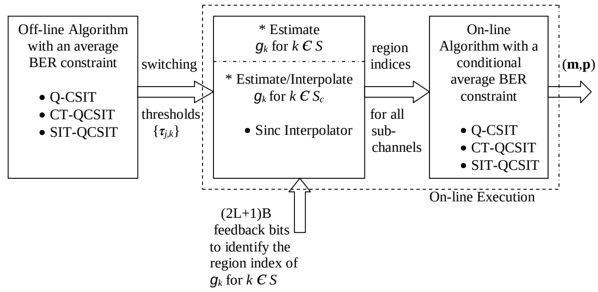

Fig. 1. Interdependence of computational modules.

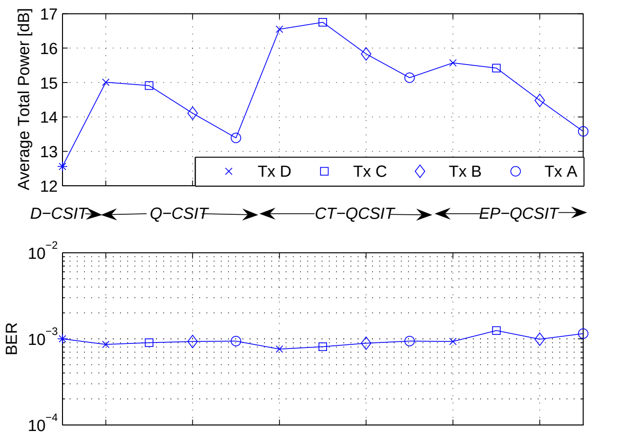

Fig. 2. Comparison of D-CSIT and Q-CSIT schemes ($N = 64$, $\text{BER}_0 = 10^{-3}$, $R_0 = 20$, $B = 2$).

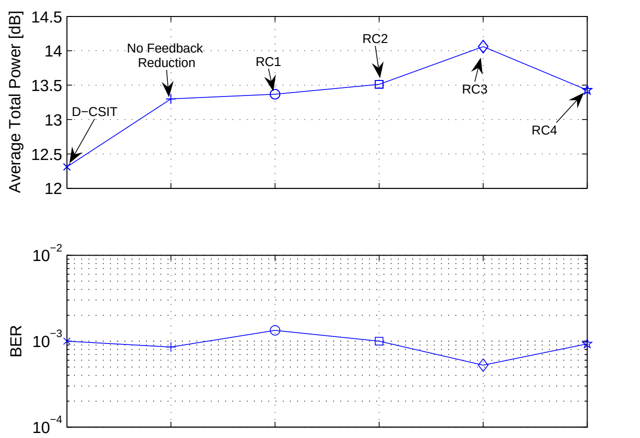

Fig. 3. Sinc interpolator with different estimation methods ($N = 64$, $R_0 = 20$, $B = 2$, $L = 3$, $\mathbb{E}[|h_0|^2] = 1$, $\mathbb{E}[|h_1|^2] = 0.5$, $\mathbb{E}[|h_2|^2] = 0.25$, $\mathbb{E}[|h_3|^2] = 0.1$).