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A decomposition method for user assignment in cellular networks with orthogonal transmissions

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Abstract

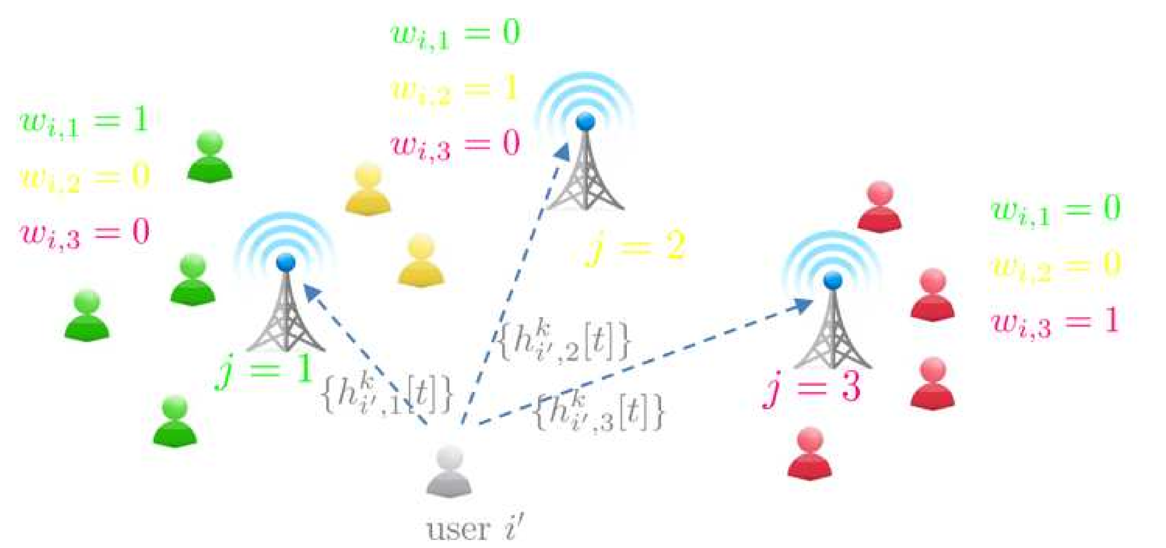
Association of users to base stations (BSs) is a classical problem. Next-generation nets require high number of BSs and quick adaptation of resources. The resources include binary variables (user-channel allocation and user-BS assignment) that render the design challenging [1, 2].

This work proposes algorithms for ***user-BS* allocation in cellular networks** where users access orthogonally and close-by BSs use non-interfering channels [3]. The *user-BS* allocation **algorithms are designed jointly** with the power, rate, and user-channel allocation.

Three different algorithms are designed, **each of them updates** (adapts) the ***user-BS* allocation at a different speed**. Linear relaxation of all the binary variables is not optimal, but a **Benders' decomposition** approach can be used to find the optimal solution.

System Model

- J BSs, I users, K channels, T time slots per horizon (indexed by j, i, k and t); $S[t]$ state
- PHY resources: power $p_{i,j}^k[t]$ and rate $r_{i,j}^k[t] = C_{i,j}(p_{i,j}^k[t], S[t])$ – REAL
- Link resources: BS-user $w_{i,j}[t]$ and channel-user $w_{i,j}^k[t]$ – BINARY
- User-BS association: fast $w_{i,j}[t]$ vs. slow $w_{i,j}$



- $\varphi_{i,j}^k[t] := \rho_{i,j}[t]C_{i,j}^k(h_{i,j}^k[t], p_{i,j}^k[t]) - \pi_{i,j}[t]p_{i,j}^k[t]$ link quality indicator, rate and power prices
- Objective: $\sum_{t=1}^T \sum_{j,i} w_{i,j}[t] \sum_k w_{i,j}^k[t]\varphi_{i,j}^k[t]$
- Constraints: $\sum_i w_{i,j}^k[t] \leq 1$, $\sum_j w_{i,j}[t] \leq 1$
- *Strong interf* and $\mathcal{K}_j$ given, but: $\mathcal{K}_j$ can depend on location, $w_{i,j}^k[t] + w_{i',j'}^k[t] \leq 1$ if conflict, ...

Prices

Variation with $t \Rightarrow$ important for quick-changing Nets
Variation with $i,j \Rightarrow$ important for heterogeneous Nets

- fixed or real-time prices set by operators
- multipliers for rate/power constraints
- marginal prices utility/cost functions (fairness) $\rho_{i,j}[t] = \dot{U}(\bar{r}_i[t])$
- state variables (congestion, battery levels) $\pi_{i,j}[t] = \mu B_j^{\max} - \mu B_j[t]$

I. Real-time association

Real-time user-BS association, no handover cost

$$\max_{\mathcal{X}} \; \sum_{t=1}^T \sum_{i,j,k} w_{i,j}^k[t]\varphi_{i,j}^k[t] \qquad \text{(1a)}$$

$$\text{s. to}: \sum_i w_{i,j}^k[t] \leq 1, \; \sum_j w_{i,j}[t] \leq 1 \qquad \text{(1b)}$$

$$w_{i,j}^k[t] = 0 \text{ if } k \notin \mathcal{K}_j, \; w_{i,j}^k[t] \leq w_{i,j}[t]$$

$$p_{i,j}^k[t] \in [0, p_{i,j}^{\max}], \; w_{i,j}^k[t] \in \{0,1\}, \; w_{i,j}[t] \in \{0,1\}$$

Separable across t, but linear relaxation not tight

Solving I

Approach: group the variables into 3 sets and show

$$\max_{\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3} \tilde{f}_0(\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3) = \max_{\mathcal{X}_2,\mathcal{X}_3} \tilde{f}_0(\mathcal{X}_1^*(\mathcal{X}_2,\mathcal{X}_3),\mathcal{X}_2,\mathcal{X}_3)$$

$$= \max_{\mathcal{X}_2,\mathcal{X}_3} \tilde{f}_0(\mathcal{X}_1^*,\mathcal{X}_2,\mathcal{X}_3) = \max_{\mathcal{X}_3} \tilde{f}_0(\mathcal{X}_1^*,\mathcal{X}_2^*(\mathcal{X}_1^*,\mathcal{X}_3),\mathcal{X}_3)$$

Power allocation*: $\mathcal{X}_1$. $\{p_{i,j}^{k*}[t]\}$ do not depend on $\{w_{i,j}^{k*}[t], w_{i,j}^*[t]\}$ and can be found as

$$p_{i,j}^{k*}[t] = \arg \max_{0 \leq p \leq p_{i,j}^{\max}} \rho_{i,j}[t]C_{i,j}(h_{i,j}^k[t], p) - \pi_{i,j}[t]p$$

Channel-user assignment: $\mathcal{X}_2$. If $w_{i,j}[t] \in \{0,1\}$, $p_{i,j}^k[t] = p_{i,j}^{k*}[t]$ given, then $w_{i,j}^{k*}[t]$ are found as:

i) Define set of winner users

$$\mathcal{I}_j^k[t] := \{i : w_{i,j}[t] = 1 \;\&\; \varphi_{i,j}^{k*}[t] = \max_l \varphi_{l,j}^{k*}[t]w_{l,j}[t]\}$$

ii) Select one user from $\mathcal{I}_j^k[t]$, call it i', and then set $w_{i',j}^{k*}[t]$=1 and $w_{i,j}^{k*}[t]$=0 for all $i \neq i'$.

User-BS assignment: $\mathcal{X}_3$. Complexity to solve for $p_{i,j}^k[t]$ and $w_{i,j}^k[t]$ polynomial. Main burden $w_{i,j}[t]$, which is MIP with IJ binary and IJK real variables. Optimal vs. approximate.

II. Slow association

User-BS association fixed during T slots

$$\max_{\mathcal{X}'} \; \sum_{t=1}^T \sum_{i,j,k} \mathbb{E}\left[w_{i,j}^k[t]\varphi_{i,j}^k[t]\right] \qquad \text{(2a)}$$

$$\text{s. to}: \text{(1b) with } \sum_j w_{i,j} \leq 1, \; w_{i,j}^k[t] \leq w_{i,j} \qquad \text{(2b)}$$

Not separable across t (only if $w_{i,j}$ given) $\Rightarrow$ MIP solvers: IJ binary and $IJKT$ real variables.

Solving II

Approach: Benders' decomposition [4].
Key idea: Master problem (difficult variables) and sub-problem (difficult ones given).

Master problem:

$$\{w_{i,j}^{(l)}\} = \arg\max_{\{z, w_{i,j}\}} z \qquad \text{(3a)}$$

$$\text{s. to}: \sum_i w_{i,j} \leq 1, \quad w_{i,j} \in \{0,1\} \qquad \text{(3b)}$$

$$z \leq \sum_{t,j,k} \left[\beta_{t,j,k}^{(l')} + \sum_i \Omega_{t,i,j,k}^{(l')} w_{i,j}\right], \quad l' < l$$

$\beta_{t,j,k}^{(l)}$ $\Omega_{t,i,j,k}^{(l)}$ — $w_{i,j}^{(l)}$ — If $w_{i,j}^{(l-1)} = w_{i,j}^{(l)}$, stop.

Subproblem:
Solve (1) w.r.t. $\{p_{i,j}^{k*}[t], w_{i,j}^{k*}[t]\}$ with $w_{i,j} = w_{i,j}^{(l)}$ given.
Store the multipliers $\beta_{t,j,k}^{(l)}$ and $\Omega_{t,i,j,k}^{(l)}$.

$\beta_{t,j,k}^{(l')}$ and $\Omega_{t,i,j,k}^{(l')} \Rightarrow \sum_i w_{i,j}^k[t] \leq 1 \quad w_{i,j}^k[t] \leq w_{i,j}^{(l')}$

III. Penalized real-time

Augment (1a) with $-\sum_{i,j} \lambda_{i,j}[t]s_{i,j}[t]$

- $\lambda_{i,j}[t] \Rightarrow$ cost of updating $w_{i,j}[t]$ (handover)
- $s_{i,j}[t] \in \{0,1\} \Rightarrow$ one if $w_{i,j}[t] \neq w_{i,j}[t-1]$

Approach for $s_{i,j}^*[t] \Rightarrow$ same than that for $w_{i,j}^*[t]$
Approach for $\lambda_{ij}[t] \Rightarrow \lambda_{i,j}[t+1] = \lambda_{i,j}[t] + \mu(s_{i,j}[t] - \eta)$

Extensions

- Stochastic and accelerated Benders
- Soft interference: interference cost $\theta_{j'}^k[t]$

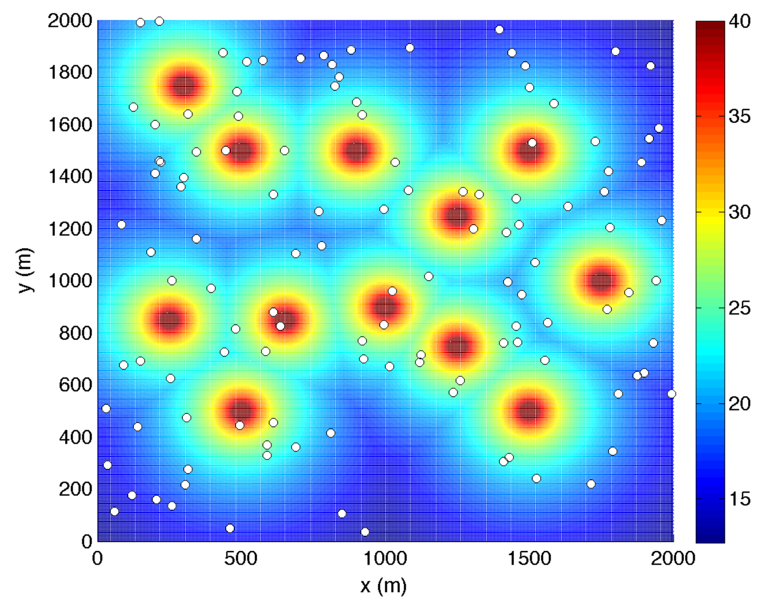
$$\tilde{\varphi}_{i,j}^k[t] = \varphi_{i,j}^k[t] - \sum_{j' \neq j} \theta_{j'}^k[t]h_{i,j'}^k[t]p_{i,j}^k[t]$$

* True also for multiple antennas.

Results

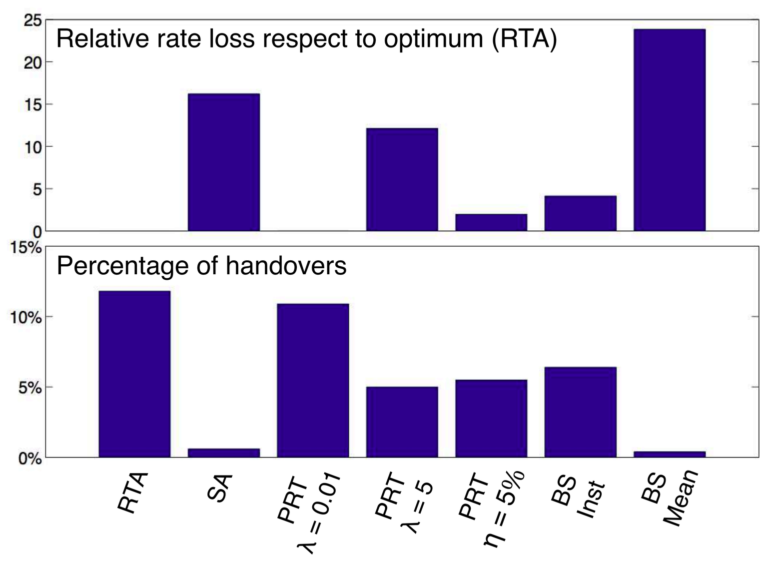
System setup:

- 120 randomly deployed users, 12 BSs, 32 i.i.d chans.
- Channel: free-space + block fading Rayleigh.
- Planning horizon: $T = 100$ (30 horizons).
- User velocity: 1 tile per scheduling period.
- Random prices: $\rho_i \in \{0.1, 0.2\}$, $\pi_j = 0.6$ for 4 macro stations, and $\pi_j = 2.4$ for 8 pico stations.



System setup. Color represents mean SNR. White dots represent user locations.

Benchmarking: Real time association (RTA), slow association (SA), penalized real time (PRT) with $\lambda = 0.01$, $\lambda = 5$ and $\eta = 5\%$, best server based on instantaneous SNR (BS Inst), best server based on mean SNR (BS Mean).



Results: relative loss respect to optimum (upper plot) and percentage of handovers (lower plot) for the seven considered algorithms.

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