A Stochastic Approximation Approach to Load Shedding in Power Networks
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ABSTRACT
A system comprising a utility company serving a set of electricity end-users is considered. The utility company can purchase energy from the wholesale market. It is also connected to a renewable energy production facility, from which it can harvest energy at no cost, and also to a battery for energy storage. Ahead of a scheduling horizon, the utility purchases energy based on forecasted demand and renewable energy production. During online operation, if the renewable energy is not adequate, real-time decisions with respect to user load shedding, energy procurement, and battery charging or discharging need to be made. The problem is cast in a stochastic approximation framework, and is solved online via a dual stochastic subgradient method with low per-slot complexity.

PROBLEM FORMULATION

Minimize
\[ \sum_{k=1}^{K} J_k(\bar{s}_k) + \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} a_k b_t^2 \] (2a)

Subject to
Time-average constraints
\[ \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} s_k(t) = \bar{s}_k, \quad k = 1, \ldots, K \] (2b)

Instantaneous constraints
\[ \pi^t - w^t \leq \sum_{k=1}^{K} s_k(t) b_t^0 + t_{\text{out}} - t_{\text{in}} \leq 0 \] (2d)

Variables: \( \{s_1, \bar{s}_1, \bar{b}_t, \{t_{\text{out}} \}, \{t_{\text{in}} \} \)

STOCHASTIC SUBGRADIENTS

- Multipliers \( \sigma = \{\sigma_k \}_{k=1}^{K} \) for (2b) and \( \rho \) for (2c)
- \( s_k^t(\sigma) = s_k^t(\sigma, \rho) \) are minimizers of the Lagrangian function given multipliers \( \sigma, \rho \), and realizations \( w, a \)
- Stochastic iterations [5], [6]: \( [x]^t = \max(0, x) \)

\[ s_k^{t+1} = \left[ s_k^t - \mu s_k^t (s_k^t - s_k^t(\sigma, \rho)) \right]^+ \] (3)

\[ \rho^{t+1} = \left[ \rho^t - \mu (\rho_{\max} - \rho_{\max}^t) \right]^+ \] (4)

Alternative version, with constant C dependent on R:
\[ \rho^{t+1} = \left[ C - \mu \rho^t \right]^+ \] (5)

SYSTEM MODEL

- Set of users \( \{1, \ldots, K \} \); set of slots \( \{1, \ldots, T \} \)
- Utility company (load-serving entity, LSE) can use renewable energy and the battery (at zero cost), and purchased energy from the market
- \( \pi^t \): actual demand – projected energy at slot \( t \)
- \( w^t \): produced renewable energy at slot \( t \)
- \( \pi^t - w^t \): real-time energy shortage, that has to be provided by \( s_k \) shed of user \( k \); \( \pi^t \) energy \( t_{\text{out}} \) from the battery, and \( \pi^t \) energy \( t_{\text{in}} \) purchased in real time at price \( a \)
- \( w, a \): stationary and ergodic \( \pi \) deterministic
- Cost \( J_k(\bar{s}_k) \) for user compensation and fairness; convex and strictly increasing

BATTERY DYNAMICS

- \( r^t \): energy stored in the beginning of slot \( t \)
- \( R \): battery capacity; \( [x]^t = \max(-x, 0) \)
- ESE charges the battery only if \( \pi^t - w^t \leq 0 \)
- \( e_{\text{in}}^t \): energy stored in the battery at slot \( t \)

\[ r^{t+1} = r^t + e^t_{\text{in}} - e^t_{\text{out}}, \quad 0 \leq r^t \leq R \] (1)

\[ 0 \leq e^t_{\text{in}} \leq \max(x, 0), \quad 0 \leq e^t_{\text{out}} \leq \min\{e^t_{\text{in}}, \{x^t - w^t\}^-\} \]

REFERENCES