Microgrid Dispatch and Price of Reliability Using Stochastic Approximation

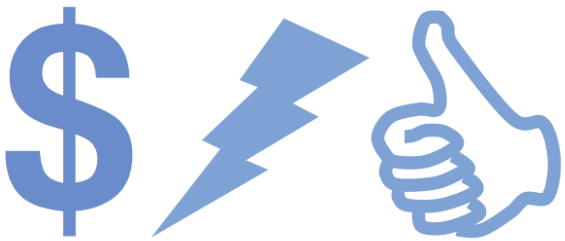
Luis M. López-Ramos[(1)], Vassilis Kekatos[(2)], Antonio G. Marques[(2)], and Georgios B. Giannakis[(3)]

[(1)] *King Juan Carlos University (Madrid, Spain)*
[(2)] *Virginia Tech. (Blacksburg, VA)*
[(3)] *University of Minnesota (Minneapolis, MN)*

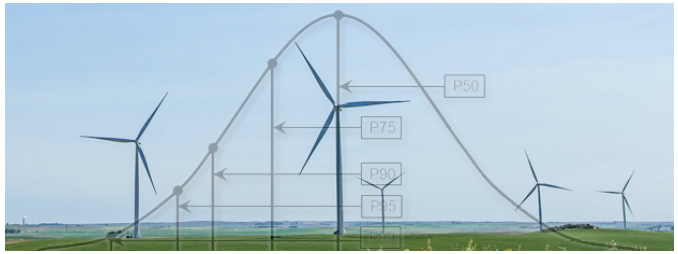
GlobalSIP'15, Orlando, FL, Dec. 2015.

Ack: A. J. Conejo , GlobalSIP Travel Grant, FPU #AP2010-1050, MEC #TEC2013-41604-R.

Motivation



- Grid efficiency vs. supply reliability
- Multi-stage economic dispatch
  - Day-ahead dispatch/scheduling
  - Recourse actions: adjustments, re-dispatching
  - Emergency actions: load shedding



- **Uncertainty** in: demand, renewable generation, market prices
- Worst-case (robust) vs. Stochastic approaches
  - Worst-case: simpler, but more conservative
  - Stochastic: non-negligible savings, maintaining reliability
- Minimize *expected* network operation cost
- Provide reliability *guarantees*

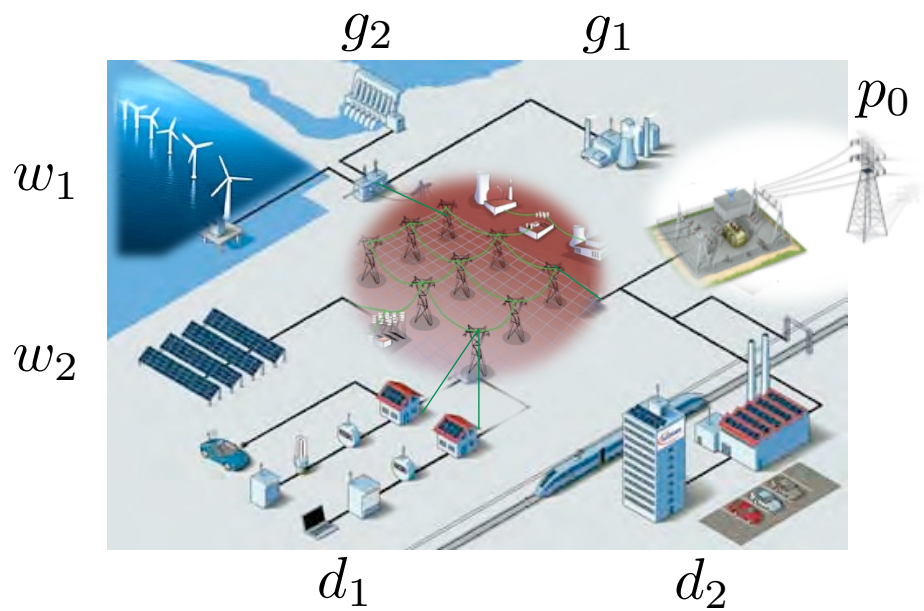
Work in context

- Bouffard and Galiana, 2008: minimization of expected costs of:
  - Preventive and Post-disturbance actions
  - Limits to *loss-of-load probability* (LOLP)
- Risk-limiting dispatch
  - Varaiya et al, 2011
  - Rajagopal et al, 2013: dynamic programming approach (curse of dimensionality)
  - Zhang et al, 2014: network constraints, price of *uncertainty*
- Our contribution:
  - Minimization of expected network cost (including penalties)
  - Limits to the expected load not served (ELNS)
  - Stochastic approximation approach

Operational Scenario

- Microgrid with:
  - Distributed Renewable energy sources
  - Interconnection with an external system where power can be sold/bought
  - Loads demanding reliable service
  - Conventional dispatchable generation

- Day-ahead dispatch:
  - Conventional dispatch
  - Market transaction
- Real-time adjustments:
  - Load shedding
  - Renewable energy spilling
  - Real-time market transaction



Grid modeling

- Linearized "DC" power flow approximation

bus power injections

renewable generation

$$\mathbf{p} = \mathbf{g} + \mathbf{w} - \mathbf{d}$$

conventional generation

load demand

$$\mathbf{1}^\top \mathbf{p} = \mathbf{0}$$

nodal injection balance

$$|\mathbf{H}^\top \mathbf{p}| \leq \mathbf{f}^{max}$$

network power transfer distribution factors

thermal power flow limits

Problem statement

Day-ahead dispatch

- **Given** *statistical information* on future network variables:
  - Load demand
  - Market prices
  - Renewable generation
- **Design** :
  - Conventional generator dispatch
  - Energy transaction with external system

Real-time dispatch

- Once day-ahead dispatch is implemented, and **given**:
  - Actual load demand
  - Real-time market prices
  - Available renewable energy
- **Design**:
  - Real-time adjustments
  - Load shedding
  - Renewable energy splilling

- **Common objective**: minimize expected operation cost
- Maintain ELNS (expected load not served) under a prespecified limit

Problem formulation

Real-time transaction cost

Load-shedding penalty

Conventional redispatch cost

Spilling penalty

$$\min_{p_0, \mathbf{g}, \{\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi})\}} \quad \mathbb{E}_{\boldsymbol{\xi}} \left[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w \right]$$

Generation dispatch cost

$$+ C(\mathbf{g}) + \beta p_0$$

Day-ahead transaction cost

Instantaneous constraints

$$\begin{aligned}
\text{s.to} \quad & \mathbf{p} = \mathbf{g} + \boldsymbol{\delta}_g + \mathbf{w} - \boldsymbol{\delta}_w - \mathbf{d} + \boldsymbol{\delta}_d \\
& \mathbf{1}^\top \mathbf{p} + p_0 + \delta_0 = 0 \\
& |\mathbf{H}\mathbf{p}| \leq \mathbf{f}^{\max} \\
& \mathbf{0} \leq \boldsymbol{\delta}_w \leq \mathbf{w} \\
& \mathbf{0} \leq \boldsymbol{\delta}_d \leq \mathbf{d} \\
& \mathbf{0} \leq \boldsymbol{\delta}_g \leq \boldsymbol{\delta}_g^{\max} \\
& \mathbf{0} \leq \mathbf{g} \leq \mathbf{g}^{\max}
\end{aligned}$$

Upper bound on ELNS

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta.$$

- This problem will be solved in two phases (no loss of optimality)

Second-stage optimization

- Suppose first-stage variables are fixed and given:

$$f(p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \ \mathbb{E}_{\boldsymbol{\xi}}[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w]$$

$$\text{s.to} \ \ (\text{inst. constraints}) \ \forall \boldsymbol{\xi};$$

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta$$

Optimal value of real-time dispatch

Couples across realizations of $\boldsymbol{\xi}$

- Solve using dual approach:

$$\mathscr{D}(\nu; p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \ \mathbb{E}_{\boldsymbol{\xi}}\left[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w + \nu(\mathbf{1}^\top \boldsymbol{\delta}_d - \eta)\right]$$

$$\text{s.to} \ \ (\text{inst. constraints}) \ \forall \boldsymbol{\xi}$$

- If ν^* is known, the problem decomposes across realizations of $\boldsymbol{\xi}$

- Stochastic dual subgradient asymptotically converges to ν^*

$$\nu^{k+1} := \left[\nu^k + \mu_k\left(\mathbf{1}^\top \boldsymbol{\delta}_d{}^*(\boldsymbol{\xi}_k; \nu^k, p_0, \mathbf{g}) - \eta\right)\right]_+$$

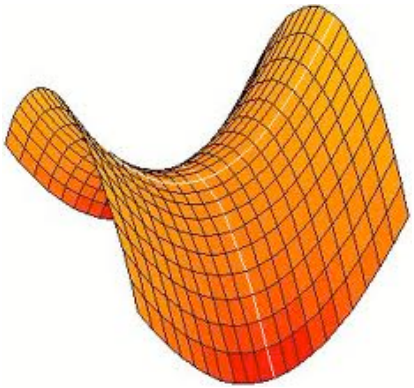
First-stage optimization

- Use info from second-stage solution to solve for $(p_0^*, \mathbf{g}^*)$

- Strong duality: $f(p_0, \mathbf{g}) = \max_{\nu} \mathscr{D}(\nu; p_0, \mathbf{g})$

$$\min_{p_0, \mathbf{g} \in \mathcal{G}} C(\mathbf{g}) + \beta p_0, + f(p_0, \mathbf{g}) = \min_{p_0, \mathbf{g} \in \mathcal{G}} \max_{\nu \geq 0} \; C(\mathbf{g}) + \beta p_0 + \mathscr{D}(\nu; p_0, \mathbf{g})$$

- Stochastic saddle point problem
- [Nemirovski et al, 2012]: *Robust stochastic approximation approach to stochastic programming*
- Solution via **stochastic saddle-point mirror algorithm**



Primal-dual subgradient iterations

- Dual update:

$$\nu^{k+1} := \left[\nu^k + \mu_k\left(\mathbf{1}^\top \boldsymbol{\delta}_d{}^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k) - \eta\right)\right]_+$$

- Primal update:

LMP at interconnection node

$$p_0^{k+1} := p_0^k - \varepsilon_k\left(\beta - \lambda^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\right)$$

$$\mathbf{g}^{k+1} := \left[\mathbf{g}^k - \varepsilon_k\left(\partial_{\mathbf{g}} C(\mathbf{g}^k) + \boldsymbol{\theta}^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\right)\right]_{\mathcal{G}}$$

LMPs at nodes with conv. gen.

- Features:
  - Derivative-free (numerical computation of LMs)
  - Distribution-free
  - Asymptotic convergence

Economic Interpretation

LMP at interconnection node

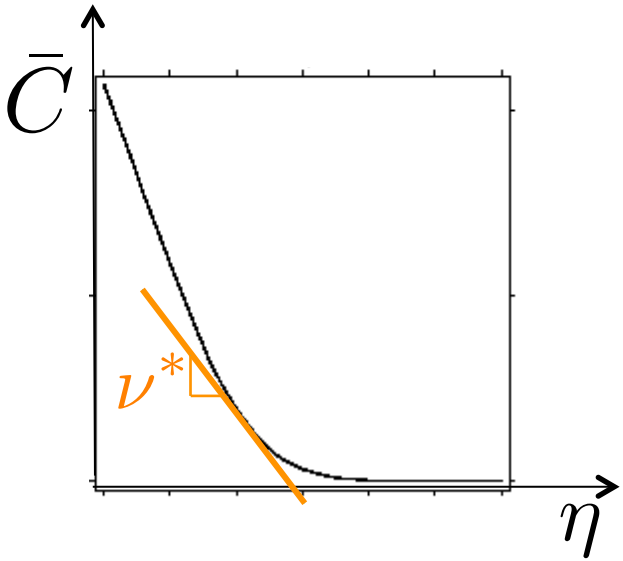
- Optimality condition: $\mathbb{E}\left[\lambda^*(\boldsymbol{\xi}_k; \nu^*, p_0^*, \mathbf{g}^*)\right] = \beta$

Day-ahead energy price

- Average price equilibrium

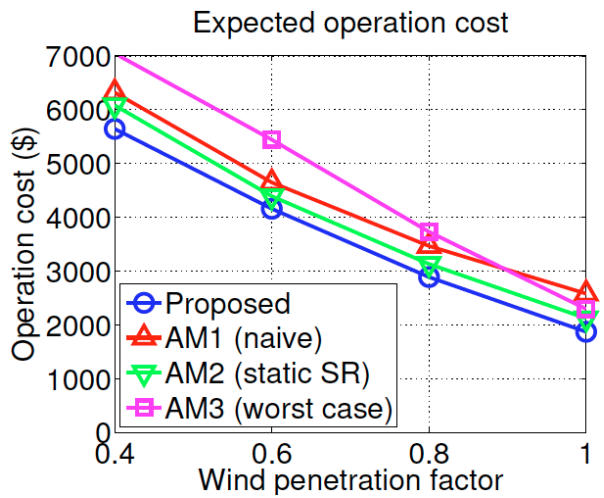
- Multiplier ν^* is the *price of reliability*
  - Sensitivity of optimal cost w.r.t. η

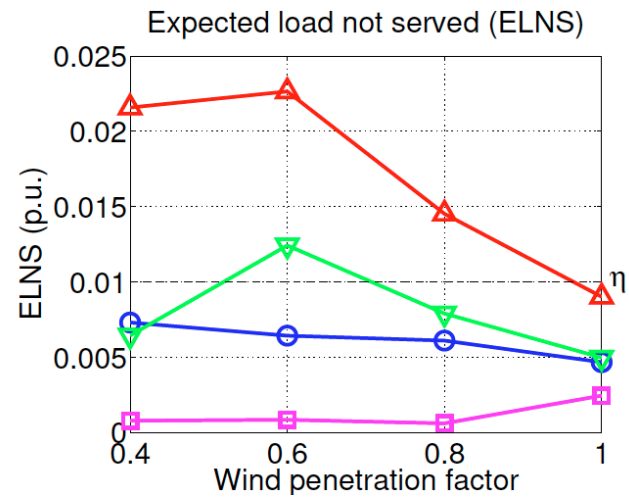
Upper bound on ELNS

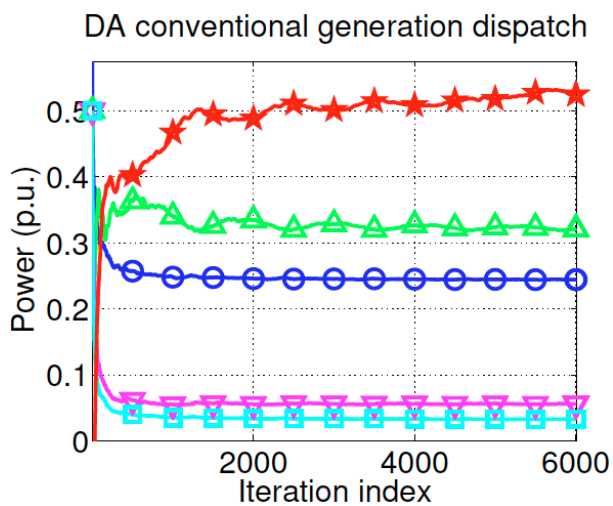


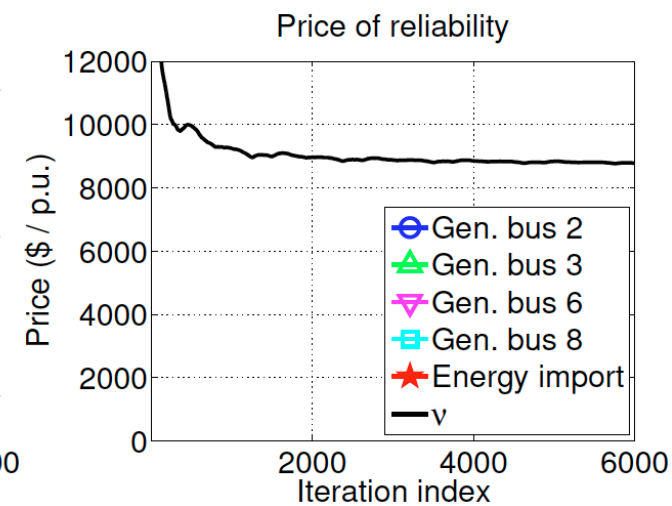
Numerical tests

- Proposed scheme:
  - Fulfills constraints
  - Outperforms alternatives
- Worst-case:
  - Over-conservative
- Static spinning reserve (SR) allocation:
  - Does not control ELNS constraint









Conclusions

- Day-ahead (DA) and Real-time (RT) dispatch
- Two-stage stochastic program
  - Constraints on expectation of second-stage design variables
- Stochastic approximation approach
  - Sample-based: efficient, distribution-free
  - Asymptotically optimal
- Future work:
  - Probability constraints (non-convex)
  - Smaller time scales: voltage control

Thank you!