Microgrid Dispatch and Price of Reliability Using Stochastic Approximation

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Ack: A. J. Conejo, GlobalSIP Travel Grant, FPU #AP2010-1050, MEC #TEC2013-41604-R.
Motivation

- Grid efficiency vs. supply reliability
- **Multi-stage** economic dispatch
  - Day-ahead dispatch/scheduling
  - Recourse actions: adjustments, re-dispatching
  - Emergency actions: load shedding
- **Uncertainty** in: demand, renewable generation, market prices
- Worst-case (robust) vs. Stochastic approaches
  - Worst-case: simpler, but more conservative
  - Stochastic: non-negligible savings, maintaining reliability
- Minimize *expected* network operation cost
- Provide **reliability guarantees**
Work in context

• Bouffard and Galiana, 2008: minimization of expected costs of:
  • Preventive and Post-disturbance actions
  • Limits to loss-of-load probability (LOLP)

• Risk-limiting dispatch
  • Varaiya et al, 2011
  • Rajagopal et al, 2013: dynamic programming approach (curse of dimensionality)
  • Zhang et al, 2014: network constraints, price of uncertainty

• Our contribution:
  • Minimization of expected network cost (including penalties)
  • Limits to the expected load not served (ELNS)
  • Stochastic approximation approach
Operational Scenario

- Microgrid with:
  - Distributed Renewable energy sources
  - Interconnection with an external system where power can be sold/bought

- Day-ahead dispatch:
  - Conventional dispatch
  - Market transaction

- Real-time adjustments:
  - Load shedding
  - Renewable energy spilling
  - Real-time market transaction

- Loads demanding reliable service
- Conventional dispatchable generation
Grid modeling

- Linearized “DC” power flow approximation

\[ \mathbf{p} = \mathbf{g} + \mathbf{w} - \mathbf{d} \]

- Bus power injections
- Renewable generation
- Conventional generation
- Load demand

| \( \mathbf{1}^{\top} \mathbf{p} = 0 \) | Nodal injection balance

| \( |\mathbf{H}^{\top} \mathbf{p}| \leq f^{\text{max}} \) | Thermal power flow limits

- Network power transfer
- Distribution factors
Problem statement

Day-ahead dispatch

- **Given** statistical information on future network variables:
  - Load demand
  - Market prices
  - Renewable generation

- **Design:**
  - Conventional generator dispatch
  - Energy transaction with external system

Real-time dispatch

- Once day-ahead dispatch is implemented, and **given**:
  - Actual load demand
  - Real-time market prices
  - Available renewable energy

- **Design:**
  - Real-time adjustments
  - Load shedding
  - Renewable energy splilling

- **Common objective:** minimize expected operation cost
- Maintain ELNS (expected load not served) under a prespecified limit
Problem formulation

\[ \min_{p_0, g, \{\delta(\xi), p(\xi)\}} \mathbb{E}_\xi \left[ R(\delta_g) + T(\delta_0) + P(\delta_d) + v_w^T \delta_w \right] \]

Real-time transaction cost  \quad Load-shedding penalty  \quad Spilling penalty

Conventional redispatch cost

Generation dispatch cost  \quad \rightarrow \quad + C(g) + \beta p_0  \quad \leftarrow \quad Day-ahead transaction cost

s.t.  \quad p = g + \delta_g + w - \delta_w - d + \delta_d

\[ 1^T p + p_0 + \delta_0 = 0 \]

\[ |H p| \leq f_{\max} \]

\[ 0 \leq \delta_w \leq w \]

\[ 0 \leq \delta_d \leq d \]

\[ 0 \leq \delta_g \leq \delta_{g_{\max}} \]

\[ 0 \leq g \leq g_{\max} \]

\[ \mathbb{E}_\xi [1^T \delta_d] \leq \eta \]

**Instantaneous constraints**

**Upper bound on ELNS**

*This problem will be solved in two phases (no loss of optimality)*
Second-stage optimization

- Suppose first-stage variables are fixed and given:

\[ f(p_0, g) := \min_{\{\delta, p\}} \mathbb{E}_{\xi}[R(\delta_g) + T(\delta_0) + P(\delta_d) + \mathbf{v}_w^T\delta_w] \]

Optimal value of real-time dispatch

\[ \mathbb{E}_{\xi}[\mathbf{1}^T\delta_d] \leq \eta \quad \text{s.to (inst. constraints) } \forall \xi; \]

- Solve using dual approach:

\[ \mathcal{D}(\nu; p_0, g) := \min_{\{\delta, p\}} \mathbb{E}_{\xi}[R(\delta_g) + T(\delta_0) + P(\delta_d) + \mathbf{v}_w^T\delta_w + \nu(\mathbf{1}^T\delta_d - \eta)] \]

s.to (inst. constraints) \forall \xi

- If \( \nu^* \) is known, the problem decomposes across realizations of \( \xi \)

- Stochastic dual subgradient asymptotically converges to \( \nu^* \)

\[ \nu^{k+1} := \left[ \nu^k + \mu_k (\mathbf{1}^T\delta_d^*(\xi_k; \nu^k, p_0, g) - \eta) \right]_+ \]
First-stage optimization

- Use info from second-stage solution to solve for \((p_0^*, g^*)\)

- Strong duality: \(f(p_0, g) = \max_{\nu} \mathcal{D}(\nu; p_0, g)\)

\[
\min_{p_0, g \in \mathcal{G}} C(g) + \beta p_0 + f(p_0, g) = \min_{p_0, g \in \mathcal{G}} \max_{\nu \geq 0} C(g) + \beta p_0 + \mathcal{D}(\nu; p_0, g)
\]

- Stochastic saddle point problem

- [Nemirovski et al, 2012]: Robust stochastic approximation approach to stochastic programming

- Solution via stochastic saddle-point mirror algorithm
Primal-dual subgradient iterations

• Dual update:

\[ \nu^{k+1} := [\nu^k + \mu_k (1^\top \delta d^*(\xi_k; \nu^k, p_0^k, g^k) - \eta)]_+ \]

• Primal update:

\[ p_0^{k+1} := p_0^k - \varepsilon_k (\beta - \lambda^* (\xi_k; \nu^k, p_0^k, g^k)) \]

\[ g^{k+1} := [g^k - \varepsilon_k (\partial_{\!\!g} C(g^k) + \theta^* (\xi_k; \nu^k, p_0^k, g^k))]_G \]

• Features:
  • Derivative-free (numerical computation of LMs)
  • Distribution-free
  • Asymptotic convergence

LMP at interconnection node
LMPs at nodes with conv. gen.
Economic Interpretation

- Optimality condition: \( \mathbb{E}[\lambda^*(\xi_k; \nu^*, p_0^*, g^*)] = \beta \)

- Average price equilibrium

- Multiplier \( \nu^* \) is the *price of reliability*
  - Sensitivity of optimal cost w.r.t. \( \eta \)

Upper bound on ELNS

LMP at interconnection node

Day-ahead energy price

Upper bound on ELNS
Numerical tests

- Proposed scheme:
  - Fulfills constraints
  - Outperforms alternatives

- Worst-case:
  - Over-conservative

- Static spinning reserve (SR) allocation:
  - Does not control ELNS constraint
Conclusions

• Day-ahead (DA) and Real-time (RT) dispatch

• Two-stage stochastic program
  • Constraints on expectation of second-stage design variables

• Stochastic approximation approach
  • Sample-based: efficient, distribution-free
  • Asymptotically optimal

• Future work:
  • Probability constraints (non-convex)
  • Smaller time scales: voltage control

Thank you!