OPTIMAL STOCHASTIC DUAL RESOURCE ALLOCATION FOR COGNITIVE RADIOS BASED ON QUANTIZED CSI*

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ABSTRACT

The present paper deals with dynamic resource management based on quantized channel state information (CSI) for multi-carrier cognitive radio networks comprising primary and secondary wireless users. For each subcarrier, users rely on adaptive modulation, coding and power modes that they select in accordance with the limited-rate feedback they receive from the access point. The access point uses CSI to maximize the sum of generic concave utilities of the individual average rates in the network while respecting rate and power constraints on the primary and secondary users. Using a stochastic dual approach, optimum dual prices are found to optimally allocate resources across users per channel realization without requiring knowledge of the channel distribution.

Index Terms— Resource management, Multiuser channels, Optimization methods, Stochastic approximation, Quantization

1. INTRODUCTION

The proliferation of wireless services along with spectrum underutilization have motivated recent research on dynamic spectrum management and wireless cognitive radios (CR) which are capable of sensing and accessing the spectrum dynamically. A number of challenges arise with such dynamic and hierarchical means of accessing the spectrum. The present paper investigates resource allocation (RA) based on quantized (Q-) CSI for CR operating over fading channels with unknown channel statistics. The focus is on a CR where co-existing primary (licensed) and secondary users [5] rely on orthogonal frequency-division multiple access (OFDMA)[1]. For such a scenario, the access point (AP) or central unit (CU) relies on the current CSI, and user specifications to optimally allocate resources and notify users about the optimal schedule through a limited-rate feedback channel. This allows users to adapt their transmissions (power, rate, and subchannel) while maximizing a given utility, respecting possible hierarchies, and adhering to power constraints and diverse quality of service (QoS) requirements.

Specifically, channel-adaptive resource (power, rate, subcarrier) allocation is obtained as the solution of a convex constrained optimization problem, which naturally takes into account different user priorities, specific utility functions, individual QoS requirements, and physical layer parameters. The resultant optimum resource allocation depends on only the current channel realization and dual variables that can be readily interpreted as user-specific prices. While the resource allocation is found in closed-form, the user-specific prices (which capture the differences among users in terms of priority, QoS as well as average channel conditions) have to be numerically found. Our focus in this paper is when channel statistics are unknown. Specifically, we develop an adaptive stochastic algorithm capable of learning the intended channels on-the-fly and converging in probability to the optimal solution.

2. MODELING PRELIMINARIES

Consider an OFDMA air interface between an AP with central scheduler and J wireless users, where users $j = 1, \ldots, J_p$ are primary spectrum holders and users $j = J_p + 1, \ldots, J$ are secondary users. The overall bandwidth B is divided into K orthogonal narrow-band subcarriers, each with bandwidth B/K small enough to ensure that the fading channel on it is flat. The wireless link between the AP and user j at subcarrier $k = 1, \ldots, K$ is characterized by its random square magnitude $h_{j,k}$ which is assumed normalized by the receiver noise variance. The resultant $JK \times 1$ vector $\mathbf{h} := \{h_{j,k},\ j = 1, \ldots, J,\ k = 1, \ldots, K\}$ is stationary and ergodic.

Per subcarrier k, we introduce a non-negative time-sharing vector $\boldsymbol{\tau}_k(\mathbf{h}) := \{\tau_{j,k}(\mathbf{h}),\ j = 1, \ldots, J\}$, where entries $\tau_{j,k}(\mathbf{h})$ depend on the channel realization $\mathbf{h}$ and obey the constraint $\sum_{j=1}^{J} \tau_{j,k}(\mathbf{h}) \in [0, 1]$. Specifically, $\tau_{j,k}(\mathbf{h})$ represents the percentage (of time) that user j gains access to subcarrier k (on the average across realizations of $\mathbf{h}$). If scheduled, i.e., if $\tau_{j,k}(\mathbf{h}) > 0$, user j transmits on subcarrier k with rate $r_{j,k}(\mathbf{h})$ and power $p_{j,k}(\mathbf{h})$.

The AP acquires with a sufficient number of training symbols the CSI vector $\mathbf{h}$ based on which it optimizes resource allocation ($\tau_{j,k}(\mathbf{h}), r_{j,k}(\mathbf{h}), p_{j,k}(\mathbf{h})\ \forall\, j, k$) and feeds back the optimal schedule to the users using a finite number of bits. This limited-rate feedback enables channel-adaptive operation based on a finite number of possible transmit-configurations. Let $\mathcal{S}$ denote a set containing a finite number of adaptive modulation, coding, and power (AMCP) modes [2], [4]. Specifically, let the mth AMCP mode for the jth user on subcarrier k consist of a discrete rate (modulation and coding) $r_{j,m,k}$; and a discrete power level $p_{j,m,k}$. Therefore, the set of AMCP modes is defined as $\mathcal{S} := \{(r_{j,m,k}, p_{j,m,k})\ |j = 1, \ldots, J,\ m = 1, \ldots, M_{j,k},\ k = 1, \ldots, K\}$, where $m = 1, \ldots, M_{j,k}$. For convenience, we extend the definition of $\mathcal{S}$ to include a fictitious user $j = 0$ with AMCP modes $(r_{0,m,k} = 0, p_{0,m,k} = 0)$ and $M_{0,k} = 1\ \forall k$ representing the case where no user transmits on subcarrier k. (Note that synchronization is assumed.)

To guarantee quality-of-service (QoS), reliability of the wireless links will be maintained under a *maximum* allowable BER $\check{\epsilon}_j$ per user. This can be satisfied provided that per channel realization $\mathbf{h}$ only the AMCP modes respecting the required BER are considered. For this purpose, with $\epsilon_{j,m,k}(p_{j,m}, r_{j,m}|h_{j,k})$ denoting the instantaneous BER expressed as a convex function of the channel gain, the

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[1]In principle any other orthogonal basis can be used as a set of transmit waveforms

power and rate per channel realization $\mathbf{h}$, we can define the set of user j active modes as

$$\mathcal{M}_j(h_{j,k}) := \{m : \epsilon_{j,m,k}(p_{j,m}, r_{j,m}|h_{j,k}) \leq \check{\epsilon}_j\}. \quad (1)$$

The finite cardinality of $\mathcal{S}$ does not necessarily force users to utilize transmit-rates and powers constrained to a specific AMCP mode (i.e., $r_{j,m,k}$ and $p_{j,m,k}$) since they can naturally support transmit-rates expressed as linear combinations of these AMCP modes by time-sharing their usage per subcarrier k. Specifically, using the mode m over $\zeta_{j,m,k}$ percentage of the $\tau_{j,k}$ time fraction, and letting $\tau_{j,m,k} := \zeta_{j,m,k}\tau_{j,k}$, user j can support rate $r_{j,k}(\mathbf{h}) = \sum_{m=1}^{M} \tau_{j,m,k}(\mathbf{h})r_{j,m,k}$ where clearly $\sum_{j=0}^{J}\sum_{m=1}^{M}\tau_{j,m,k} \in [0,1]$ and now the time-allocation vector is defined as $\boldsymbol{\tau}(\mathbf{h}) := \{\tau_{j,m,k}(\mathbf{h}), j = 1,\ldots,J,\ m = 1,\ldots,M_{j,k},\ k = 1,\ldots,K\}$. Through time-sharing, any linear combination of $\{r_{j,m,k}\}$ gives rise to the same linear combination of corresponding powers $\{p_{j,m,k}\}$; hence $p_{j,k}(\mathbf{h}) = \sum_{m=1}^{M}\tau_{j,m,k}(\mathbf{h})p_{j,m,k}$.

3. CHANNEL-ADAPTIVE RESOURCE ALLOCATION

The optimal resource allocation will be obtained in this section as the solution of a constrained optimization problem. The *objective* of this will be based on concave and increasing so called *utility functions* $U_j(\cdot)$, that are commonly used in resource allocation problems (not only restricted to communication systems). On the other hand, to respect primary/secondary CR hierarchies, a *minimum* average rate $\check{r}_j$ will be maintained for primary user transmissions indexed by $j \leq J_p$; while, to prevent secondary users from "abusing" the spectrum, *maximum* average rates $\check{r}_j$ will be imposed for these users too indexed by $j > J_p$. Finally, *maximum* individual average power constraints $\check{p}_j$ will be present for both primary and secondary users.

Then the optimal allocation will maximize the total utility subject to (s.to) average rate, average power and time feasibility constraints. Considering the case where the input of the utility function for the jth user (denoted by x_j) corresponds to the average rate transmitted by that user and with $E_{\mathbf{h}}[\cdot]$ denoting the expectation over $\mathbf{h}$, the optimal allocation can be found as the solution of:

$$\begin{cases} \max\limits_{\boldsymbol{\tau}(\mathbf{h}),\mathbf{x}} \ \sum_{j=0}^{J} U_j(x_j) \\ \text{s.to} \\ c1.\ E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h})r_{j,m,k}\right] \geq \check{r}_j, j \leq J_p \\ c2.\ E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h})r_{j,m,k}\right] \leq \check{r}_j, j > J_p \\ c3.\ E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h})p_{j,m,k}\right] \leq \check{p}_j,\ \forall j \\ c4.1.\ \tau_{j,m,k}(\mathbf{h}) \geq 0;\ \forall \mathbf{h},\ \forall j,m,k \\ c4.2.\ \sum_{j=1}^{J}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h}) \leq 1;\ \forall \mathbf{h},\ \forall k \\ c5.\ x_j = E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h})r_{j,m,k}\right],\ \forall j. \end{cases} \quad (2)$$

where constraints $c1$ and $c2$ enforce the primary-secondary CR hierarchies; constraints $c3$ ensure adherence to the power budget of individual users; constraints $c4$ impose that the user allocation has to be feasible; and constraints $c5$ represent the utility input[2]. The problem formulated as in (2) is convex and can be efficiently solved using a Lagrange multiplier based primal-dual approach [1]. Note that if $U_j(x_j) := \theta_j x_j$ the problem in (2) reduces to the classical "weighted" (by θ_j) rate maximization problem.

[2]If the problem in (2) is feasible, the equality ($=$) in $c5$ can be relaxed with inequality ($\leq$) without loss of optimality

3.1. Characterizing the optimum channel-adaptive RA

Let λ_{r_j} and λ_{p_j} denote the Lagrange multipliers associated with rate and power constraints of the primary ($j \leq J_p$) and secondary ($j > J_p$) users, and w_j the multiplier corresponding to $c5$. Ignoring temporarily the instantaneous constraints $c4$ and with $\bar{r}_j(\boldsymbol{\tau}) := E_{\mathbf{h}}[\sum_{k=1}^{K}\sum_{m\in\mathcal{M}_j(h_{j,k})}\tau_{j,m,k}(\mathbf{h})r_{j,m,k}]$, the Lagrangian as a function of $\boldsymbol{\lambda} := [\lambda_{r_1}, \lambda_{p_1}, \ldots, \lambda_{r_J}, \lambda_{p_J}]^T$, $\boldsymbol{w} := [w_1, \ldots, w_J]^T$, $\boldsymbol{\tau}$, and $\boldsymbol{x} := [x_1, \ldots, x_J]^T$ is given by

$$L(\boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\tau}, \boldsymbol{x}) := \sum_{j=1}^{J} U_j(x_j) + \sum_{j=1}^{J}(-1)^{\mathbf{I}_{\{j>J_p\}}}\lambda_{r_j}(\bar{r}_j(\boldsymbol{\tau}) - \sum_{j=1}^{J}\lambda_{p_j}(\bar{p}_j(\boldsymbol{\tau}) - \check{p}_j) - \sum_{j=1}^{J} w_j(x_j - \bar{r}_j(\boldsymbol{\tau})) \quad (3)$$

where $\mathbf{I}_{\{\cdot\}}$ stands for the indicator function. The Lagrange dual function is

$$D(\boldsymbol{\lambda}, \boldsymbol{w}) := \max_{\boldsymbol{x},\boldsymbol{\tau}\in\{c4.1,c4.2\}} L(\boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\tau}) \quad (4)$$

where the set of constraints $c4.1$ and $c4.2$ has to be explicitly imposed since it was not originally considered in the Lagrangian. Finally, with $\boldsymbol{\lambda} \geq \mathbf{0}$ denoting that all entries of $\boldsymbol{\lambda}$ are non-negative, the dual problem of (2) is

$$\min_{\boldsymbol{\lambda}\geq\mathbf{0},\boldsymbol{w}\geq\mathbf{0}} D(\boldsymbol{\lambda}, \boldsymbol{w}). \quad (5)$$

Since (2) has zero-duality gap, solving the problem in (5) amounts to solving the original constrained problem in (2). But to solve (5), we will need first to solve the maximization in (4). As we will see, given $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ (that soon will be interpreted as dual prices) the optimum time allocation $\tau^*_{j,m,k}(\boldsymbol{\lambda}, \mathbf{h})$ solving (4) (which depends on both the current channel realization and the value of the multipliers) can be analytically found. However, there is no closed-form for the solution of (5). Our approach in this paper will consist of first finding the expression of $\tau^*_{j,m,k}(\boldsymbol{\lambda}, \mathbf{h})$ and then finding the optimum $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ based on stochastic iterations.

To solve (4), we first define the link quality indicators

$$\varphi_{j,m,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) := \varphi_{j,m,k}(\lambda_j, w_j, h_{j,k}) := \quad (6)$$
$$(w_j + (-1)^{\mathbf{I}_{\{j>J_p\}}}\lambda_{r_j})r_{j,m,k} - \lambda_{p_j}p_{j,m,k},\ \ \forall m \in \mathcal{M}_j(h_{j,k}), \forall j$$

where by construction $\varphi_{0,m,k} = 0$. Per subcarrier k, we determine for each user j the "most-efficient" mode in the sense that

$$m^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) = \arg\max_{m\in\mathcal{M}_j(h_{j,k})} \varphi_{j,m,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}); \quad (7)$$

and select the "most-efficient" user as the one with index

$$j^*_k(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) = \arg\max_{j} \varphi_{j,m^*_{j,k},k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) \quad (8)$$

which in general is unique (remember that $\mathbf{h}$ is a random variable). For the case where the winner is unique, it can be shown that per sub-carrier k, the optimal schedule of time-sharing fractions is

$$\tau^*_{j,m,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) = \begin{cases} 1, & \text{if } j = j^*_k \text{ and } m = m^*_{j^*_k,k} \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

i.e., the "most-efficient" user is the only user gaining access to the subcarrier k; for this reason j^*_k will be termed "winner user" of subcarrier k. If eventually more than one user attains the maximum, the policy of allowing only one of them accessing the subcarrier is still optimum, although in this case the specific user selected for transmission is randomly chosen among the multiple winners so that the

QoS constraints are met with equality (if the AMCP modes are linearly independent, the probability of a tie vanishes as J, K or $M_{j,k}$ increases). The proof of (9) is omitted due to space limitations but follows the lines of [3] and [4].[3]

Based on the optimal time allocation (9), we can express the optimum transmit-rate and power per user and subcarrier as

$$r^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) := \textstyle\sum_{m\in\mathcal{M}_j(h_{j,k})} \tau^*_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) r_{j,m,k} \qquad (10)$$
$$p^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) := \textstyle\sum_{m\in\mathcal{M}_j(h_{j,k})} \tau^*_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}, \mathbf{h}) p_{j,m,k}.$$

Finally, with $U_j'^{-1}$ denoting the inverse function of U_j', the input x_j maximizing (4) can be easily found as $x^*_j(\boldsymbol{w}) = U_j'^{-1}(w_j)$.

3.2. Finding the optimal dual prices: learning the environment

Since the problem in (2) is convex, the optimum $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$ can be found by searching over the dual function which is always convex and its global optimum can be found using (sub)gradient iterations [1, Chap. 6]. However, since the constraints in (2) involve expectations over the channel gains, averaging over the PDF of the channel is needed and thus the channel statistics are in principle required. To bypass this problem, we will rely on adaptively updated *instantaneous* estimates of $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ based on stochastic implementations of the subgradient iterations [6].

To this end, suppose that the fading channel vector $\mathbf{h}$ remains invariant over a block of OFDMA symbols but can vary from block-to-block (block fading channel model). Let n denote the current block (time) index, $\mathbf{h}^{[n]}$ the fading state during block n, and $[x]^+ := \min(x, 0)$. Then, we can execute an on-line recursion across blocks to obtain the instantaneous estimates $\hat{\boldsymbol{w}}^{[n]} := [\hat{w}_1^{[n]}, \ldots, \hat{w}_J^{[n]}]^T$ and $\hat{\boldsymbol{\lambda}}^{[n]} := [\hat{\lambda}_{r_1}^{[n]}, \hat{\lambda}_{p_1}^{[n]}, \ldots, \hat{\lambda}_{r_J}^{[n]}, \hat{\lambda}_{p_J}^{[n]}]^T$ as

$$\hat{\lambda}_{r_j}^{[n+1]} = \left[\hat{\lambda}_{r_j}^{[n]} - \beta^{[n]}\left[\textstyle\sum_{k=1}^K r^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]}) - \check{r}_j\right]\right]^+ \qquad (11)$$
$$\hat{\lambda}_{r_j}^{[n+1]} = \left[\hat{\lambda}_{r_j}^{[n]} - \beta^{[n]}\left[\check{r}_j - \textstyle\sum_{k=1}^K r^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]})\right]\right]^+$$
$$\hat{\lambda}_{p_j}^{[n+1]} = \left[\hat{\lambda}_{p_j}^{[n]} - \beta^{[n]}\left[\check{p}_j - \textstyle\sum_{k=1}^K p^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]})\right]\right]^+$$
$$\hat{w}_j^{[n+1]} = \left[\hat{w}_j^{[n]} - \beta^{[n]}\left[x^*_j(\hat{\boldsymbol{w}}^{[n]}) - \textstyle\sum_{k=1}^K r^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]})\right]\right]^+$$

where $r^{*[n]}_{j,k} := r^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]})$ and $p^{*[n]}_{j,k} := p^*_{j,k}(\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]})$ are computed based on (10) and represent the current rate and power of the user j at subcarrier k over block n; and stepsize $\beta^{[n]} \in [0, 1]$ implements a forgetting factor in the averaging. To find $r^{*[n]}_{j,k}$ and $p^{*[n]}_{j,k}$ per block n, the optimum AMCP mode and user for each subcarrier k have to be found by substituting the current $\hat{\boldsymbol{\lambda}}^{[n]}, \hat{\boldsymbol{w}}^{[n]}, \mathbf{h}^{[n]}$ estimates into (6)-(9). Once the RA parameters of the nth block are obtained, we can use (11) to update both reward weights $\hat{\boldsymbol{w}}^{[n+1]}$ and dual prices $\hat{\boldsymbol{\lambda}}^{[n+1]}$ with negligible (linear) computational complexity.

[3]Regarding λ_{r_j} and λ_{p_j} as rate and power "prices" and $w_j = U_j'(x_j)$ as a rate-weight representing the marginal utility per transmitted bit, the link quality indicators in (6) determine the net rate reward (rate reward minus power cost) corresponding to the (j, m)th mode on subcarrier k. In our secondary market CR set-up, to satisfy the *individual* QoS per user we promote the marginal utility of the primary users $j \leq J_p$ through addition of the multiplier $\lambda_{r_j} > 0$; whereas these positive multipliers are subtracted from the marginal utility to prevent abusive spectrum access by secondary users $j > J_p$ (likewise, $\lambda_{p_j} > 0$ can be always viewed as a penalty or cost and $w_j > 0$ as a reward).

Per block n, this algorithm performs a weighted sum-rate maximization with adaptive weights provided by $\hat{w}_j^{[n]} + \hat{\lambda}_j^{[n]}$ for primary users and $\hat{w}_j^{[n]} - \hat{\lambda}_j^{[n]}$ for secondary users to obtain on-line optimal allocation, whereas the variables $\hat{\boldsymbol{\lambda}}^{[n+1]}$ and $\hat{\boldsymbol{w}}^{[n+1]}$ are updated using *instantaneous* transmit-powers and rates.

This simple stochastic dual (SD) on-line algorithm can learn the channel statistics on-the-fly, and is convergent and asymptotically optimal as the following theorem states.

Theorem 1 *If problem (2) is strictly feasible, then the estimates obtained recursively in (11) using any initial* $\hat{\boldsymbol{\lambda}}[0] \geq \mathbf{0}$ *and* $\hat{\boldsymbol{w}}[0] \geq \mathbf{0}$*, converge in probability to the optimal* $\boldsymbol{\lambda}^*$ *and* $\boldsymbol{w}^*(\boldsymbol{\lambda}^*)$ *of (2), as* $n \to \infty$ *and* $\beta^{[n]} \downarrow 0$.

The proof is omitted due to space limitations but it can follow the lines of the convergence proof of the queue size updates of the greedy primal-dual algorithm in [7] and requires $\sum_{n=1}^{\infty} \beta^{[n]} \to \infty$. Equally interesting, with a small but constant stepsize $\beta^{[n]} = \beta$, the SD algorithm brings $\hat{\boldsymbol{\lambda}}^{[n]}$ to a small neighborhood of $\boldsymbol{\lambda}^*$. Because this adaptive algorithm converges from arbitrary initializations it exhibits robustness to channel non-stationarities.

4. REDUCING THE FEEDBACK: QUANTIZED CSI

The optimal resource allocation presented so far can be easily implemented when the CU (scheduler) knows the price vectors $\hat{\boldsymbol{\lambda}}^{[n]}$ and $\hat{\boldsymbol{w}}^{[n]}$, the set of AMCP modes $\mathcal{S}$, and the vector channel realization $\mathbf{h}^{[n]}$. Based on those the optimal transmit configuration per subcarrier ($r^{*[n]}_{j,k}$, $p^{*[n]}_{j,k}$, $\forall j, k$) can be computed using (10) and fed back to the CR user terminals using $B = \lceil \sum_{k=1}^K \log_2(\sum_{j=0}^J M_{j,k}) \rceil$ bits per channel realization (so that per subcarrier we can index any user-mode pair in $\mathcal{S}$). However, the feedback required from the CU can be reduced with small loss of performance using channel quantization.

To perform this design we will assume that instead of the analog-valued $h_{j,k}$ (perfect (P-)CSI), the optimization algorithm relies on the quantized value $h^Q_{j,k}$ (Q-CSI). This value is found using a channel quantizer and belongs to a set $\mathcal{L}_{j,k}$ with finite cardinality $L_{j,k}$ so that we can write $\mathcal{L}_{j,k} := \{h_{j,k,l}\}_{l=1}^{L_{j,k}}$. Since the set of feasible modes $\mathcal{M}_{j,k}(h_{j,k})$ satisfying the BER constraint in (1) was selected in accordance with $h_{j,k}$, it is necessary to adapt this definition to the quantized set-up. To do so, it is first useful to introduce the function $\epsilon^Q_{j,m,k}(p_{j,m,k}, r_{j,m,k}|h^Q_{j,k})$ which expresses the BER as a convex function of the power, the rate, and the *quantized* version of the channel. Based on this function, we define the set of AMCP modes satisfying the instantaneous BER requirement $\check{\epsilon}$ as

$$\mathcal{M}^Q_{j,k}(h^Q_{j,k}) := \{m : \epsilon^Q_{j,m,k}(p_{j,m,k}, r_{j,m,k}|h^Q_{j,k} = h_{j,k,l}) \leq \check{\epsilon}\}. \qquad (12)$$

Furthermore, we will assume that the CU knows $\hat{\boldsymbol{\lambda}}^{[n]}$, $\hat{\boldsymbol{w}}^{[n]}$, $\mathcal{S}$ and each terminal j knows its own dual prices $\lambda^{[n]}_{r_j}$, $\lambda^{[n]}_{p_j}$, $w^{[n]}_j$ and AMCP modes $\mathcal{S}_j := \{(r_{j,m,k}, p_{j,m,k}), \forall m, k\}$. Then the feedback overhead can be reduced under the following operating conditions:

(oc.1) Both CU and users: (i) use the same $\hat{\boldsymbol{\lambda}}[0]$ and $\hat{\boldsymbol{w}}[0]$; (ii) identical $\beta^{[n]}$ $\forall n$; and (iii) replace $\mathcal{M}_{j,k}(h_{j,k})$ by $\mathcal{M}^Q_{j,k}(h^Q_{j,k})$ in every step of the calculations.

(oc.2) For each block index n, the CU (receiver):

(i) substitutes $\hat{\boldsymbol{\lambda}}^{[n]}$ and $\hat{\boldsymbol{w}}^{[n]}$ into (6)-(9) to find the optimal RA $\forall j, k$; (ii) runs the dual updates in (11)-(12) to obtain $\hat{\boldsymbol{\lambda}}^{[n+1]}$ and $\hat{\boldsymbol{w}}^{[n+1]}$ $\forall j, k$; and (iii) feeds back to the users the message $\mathbf{c}^{[n]} := [j_1^{*[n]},$

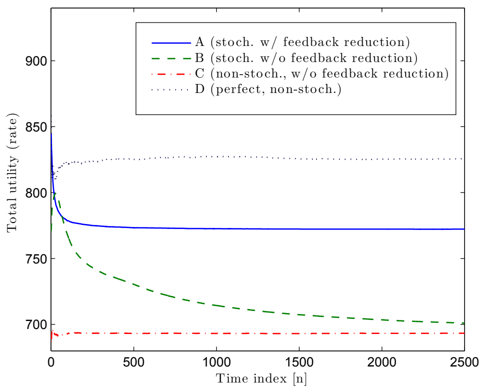

Fig. 1. Evolution of the total sum utility with time.

$l_1^{*[n]}, \ldots, j_K^{*[n]}, l_K^{*[n]},]$, where $l_k^{*[n]} := \{l : h^Q_{j_k^{*[n]},k} = h_{j_k^{*[n]},k,l}\}$, i.e., $l_k^{*[n]}$ represents the index of the quantization region the channel gain of the winner user belongs to.

(oc.3) For each block index n, the terminals (transmitters): (i) the winner terminal $j_k^{*[n]}$ (indexed by the CU) uses $h^Q_{j,k} = h_{j,k,l_k^{*[n]}}$ (indexed by the CU) plus $\lambda_{p_j}^{[n]}$, $\lambda_{\check{p}_j}^{[n]}$, and $w_j^{[n]}$ (locally stored) to find its optimum transmission mode $m_k^{*[n]}$, while all other users set their transmission power and rate on this subcarrier to zero; and (ii) once every terminal knows its transmit-rate and power $\forall k$, it updates its own $\lambda_{p_j}^{[n+1]}$, $\lambda_{\check{p}_j}^{[n+1]}$, and $w_j^{[n+1]}$ using (11).

Using this modification the rate required for the feedback link between the CU and the users reduces to $B = \lceil \sum_{k=1}^{K} \log_2 (\sum_{j=0}^{J} L_{j,k}) \rceil$ bits per $\mathbf{h}$. Note that if uplink and downlink channels are reciprocal, each user can estimate its own channel and then the CU only has to feedback the winner-user per subcarrier. Finally, if needed, on top of our quantization design further reduction of the feedback overhead can be effected by exploiting the possible correlation among channel gains across subcarriers and/or time.

5. NUMERICAL TESTS

Due to space limitations, we only show a single test case simulating an adaptive OFDMA system with 1 primary and 3 secondary users (i.e., $J = 4$), $K = 256$ subcarriers, 10 uncorrelated Rayleigh taps per user, and average signal-to-noise ratio per subcarrier equal to 6dB. The default utility is linear (i.e., $U_j(\bar{r}_j) = \bar{r}_j$ and $\boldsymbol{w} = \mathbf{1}$); the AMCP modes are designed so that they correspond to non-zero uniform random samples of the continuous water-filling solution [3], i.e., $r_{j,m,k} = [\log_2(h/\mu)]_{>0}$ and $p_{j,m,k} = [1/\mu - 1/h]_{>0}$ with the water-filling level $\mu \in [0.01, 100]$ and the channel gain $h \in [0.5, 5\bar{h}_{j,k}]$; and $M_{j,k} = 64\ \forall (j,k)$. The channel quantizer corresponds to the equally-probable quantizer presented in [4] with $L_{j,k} = 8\ \forall (j,k)$. The QoS constraints are set to: $\check{\mathbf{r}}^T := [\check{r}_1, \ldots, \check{r}_4] = [100, 50, 50, 100]$ bits per channel use (b.p.c.u), $\check{\mathbf{p}}^T := [\check{p}_1, \ldots, \check{p}_4] = [1000, 200, 1000, 200]$, and $\check{\epsilon}_j = 0.001$ for all j.

Figure 1 depicts the summation of the sample mean of the individual rates (i.e., $\sum_{j=1}^{J} (\hat{\bar{r}}_j(n))$ with $\hat{\bar{r}}_j(n) = \frac{1}{n} \sum_{k=1}^{n} r_j^{*[k]}$) as a function of n for 4 different cases. Namely: (A) the case where the dual prices are computed stochastically as in (11) and the mode set corresponding to the channel quantization in (12) is considered; (B) the case where the dual prices are computed stochastically as in (11) and the original mode set in (1) is considered; (C) the case where the dual prices are computed using the original average version of the subgradient iterations (thus, the channel PFD is known) and the mode set in (1) is considered; and (D) the benchmark case where dual prices are computed using the original average version of the subgradient iterations and the mode set in (1) is assumed of infinite size (thus the RA can even follow the water-filling solution). The main results that Figure 1 provides are: (i) the stochastic algorithms converge; (ii) the convergence is attained in a small number of iterations (as expected the convergence is slightly slower for the stochastic algorithms, since they only use past information while C and D know the PFD of the channel and therefore can anticipate future events), (iii) the utility achieved by the schemes based on finite number of transmit-configurations is close to the one of the benchmark (the power consumption is slightly higher for the stochastic schemes). Although not plotted, our numerical experiments also include a heuristic algorithm that keeps fixed the subcarrier and power allocation but optimally allocates the rate in each subcarrier. The total transmitter rate achieved by this algorithm was 25 bits (more than an order of magnitude less). Finally, it is worth mentioning that all the depicted algorithms satisfied the power and rate constraints (the power constraint was active for user 1 while the secondary rate constraints were the active ones for users 2, 3 and 4), while the heuristic design was not able to do it.

6. CONCLUSIONS

We developed an optimal channel-adaptive algorithm for allocating power, rate, and subcarriers in OFDMA cognitive radios with a primary-secondary user hierarchy and limited-rate feedback. The resultant optimum resource allocation depends on the current channel realization and optimally calculates dual prices. A provably convergent stochastic dual algorithm was developed to learn the optimum value of the dual prices on-the-fly. Once the values of the dual prices are obtained, the overall optimal solution is fairly simple and amounts to opportunistic access whereby only one user gains access to a given frequency band per channel realization. The required complexity to implement the novel algorithm and the amount of feedback are affordable for most practical systems.

7. REFERENCES

[1] D. Bertsekas, *Nonlinear Programming*, 2nd Ed., Athena Scientific, 1999.

[2] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[3] L. Li and A. J. Goldsmith, "Capacity and Optimal Resource Allocation for Fading Broadcast Channels–Part I: Ergodic Capacity," *IEEE Trans. on Inf. Theory*, vol. 47, no. 3, pp. 1083-1102, March 2001.

[4] A. G. Marques, F. F. Digham, G. B. Giannakis, and F. J. Ramos, "Reduced-Complexity Power-Efficient Wireless OFDMA using an Equally Probable CSI Quantizer", *Proc. of IEEE Intl. Conf. on Commun.*, Glasgow, Scotland, Jun. 24-28, 2007.

[5] L. M. Peha, "Approaches to Spectrum Sharing," *IEEE Commun. Magazine*, vol. 43, no. 2, pp. 10-12, Feb. 2005.

[6] P. Kall and S. W. Wallace, *Stochastic Programming*. Wiley, 1994.

[7] A. Stolyar, "Maximizing Queueing Network Utility Subject to Stability: Greedy Primal-Dual Algorithm," *Queueing Systems*, vol. 50, no. 4, pp. 401–457, Aug. 2005.