# Underlay Cognitive Radios with Finite Transmission Modes and Capacity Guarantees for Primary Users

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***Abstract*—We design adaptive resource allocation schemes for cognitive radios so that the sum-rate of secondary users is optimized while the damage (interference) to the primary users is keep under control. Secondary users transmit orthogonally and adhere to limits on: a) the long-term interfering power at each primary receiver and b) the long-term capacity loss inflicted to each primary receiver. We first analyze the single antenna case and then consider that the secondary users implement adaptive beamforming. The focus of the paper is on scenarios where users can implement only a *finite* number of power levels and beamforming vectors. Although b) renders the resultant optimization problem non-convex, it holds that it has zero duality gap and that, due to the favorable structure in the dual domain, it can be solved in polynomial time. Specifically, it holds that the computational complexity required to obtain the optimum resource allocation for a given fading realization is proportional to the number of: secondary users, primary users (channels), power levels, and beamforming vectors.**

***Index Terms*—Cognitive radios, resource management, non-linear optimization, finite-rate-feedback.**

## I. Motivation and Contribution

Spectrum scarcity and proliferation of new wireless services have motivated recent research on dynamic spectrum management and wireless cognitive radios (CRs). Secondary users (SUs) in the CR adapt their transmissions to limit the interference to the primary user (PU) receivers that hold the licence of the frequency band [1], [2]. To achieve that goal, the CR needs to sense (acquire) the channel state information (CSI) of both primary and secondary links. The information of secondary links allows SUs to mitigate fading and take advantage of good channel realizations, while the information of primary links guarantees that interference is kept under control. Based on the measurements obtained through sensing, SUs will adapt their available resources to the instantaneous channel conditions.

We focus on *underlay* CRs where SUs adapt their available resources (beamforming vectors and power loadings) dynamically, and access orthogonally a set frequency bands which are originally devoted to PU transmissions. Orthogonal here means that if a SU is transmitting, no other SU can be active in the same band. The resource allocation (RA) schemes are then obtained as the solution of a sum-rate maximization subject to limits on: a) the long-term interfering power at each PU and b) the long-term capacity loss inflicted to each PU. The resources are constrained to belong to predesigned finite-size codebooks, so that only a few bits of feedback are required to identify the optimal resource allocation. Consideration of b) is challenging because the interfering SU powers render the capacity term non-convex. Joint consideration of b) and finite-rate-feedback is the main contribution of this work. Although non-convex, it holds that the formulated problem has zero duality gap; hence, the Langrangian relaxation is optimal [3]. Moreover, the operating conditions of the secondary network are such that the problem in the dual domain can be decomposed (separated) across users, power levels, beamforming vectors, and frequency bands. This favorable structure allows for polynomial-time solution and, hence, renders the non-convex problem computationally tractable [5], [4]. To facilitate exposition, the algorithms are designed under the assumption of perfect CSI and scalar channels. Then, the multiple-antenna setup is described and the optimum adaptive beamforming is also designed. The design of the optimum RA schemes for a similar setup considering scalar channels and continuous power allocation was addressed in [5], the contribution here is the consideration of: i) finite transmission modes and ii) adaptive beamforming. The design of the power and beamforming codebooks is left as future work.

## II. Optimal scheduling and power loading

We consider a CR with $M$ SUs (indexed by $m$) transmitting opportunistically and orthogonally over $K$ different frequency bands (indexed by $k$). For simplicity, we assume that: i) each band is occupied by a different PU; and ii) the secondary network has a network controller (NC) which collects the CSI and then makes the RA decisions. The CSI at instant $n$ is denoted as $\mathbf{h}[n] := \{h_{k,1}^m[n], h_{k,2}^m[n] | \forall k, m\}$, where $h_{k,2}^m[n]$ represents the instantaneous equivalent low-pass coefficient for the $m$th SU-to-SU link at band $k$ and $h_{k,1}^m[n]$ represents the instantaneous equivalent low-pass coefficient of the channel between the $m$th SU transmitter and the $k$th PU receiver. Such coefficients are assumed to be normalized by the corresponding power noise. Moreover, $h_{k,2}^m[n]$ also accounts for the interference caused by the PU transmitters (if any).

Now, we introduce the variables to be designed (resources to be allocated): $w_{k,2}^m$ denotes a boolean variable such that $w_{k,2}^m = 1$ if the $m$th SU is scheduled to transmit into the $k$th band and $w_{k,2}^m = 0$ otherwise. Provided that $w_{k,2}^m = 1$, let $p_{k,2}^m$ denote the instantaneous power transmitted over the $k$th band by the $m$th SU. We consider a scenario where transmitters can only transmit using one out of a finite number of power levels. Let $L_P$ be the number of power levels a SU can use, $p_{k,2}^{m,l}$ the $l$th power level for the pair $(m,k)$, and let $\mathcal{P}_{k,2}^m := \{p_{k,2}^{m,l}\}_{l=1}^{L_P}$ denote the corresponding power codebook. We have then that $p_{k,2}^m \in \mathcal{P}_{k,2}^m$, with $\mathcal{P}_{k,2}^m = \mathcal{P}_{k',2}^{m'}$ in most practical systems. Under capacity constraints, the instantaneous rate and power variables are coupled through the function $r_{k,2}^m(x) := \log_2(1+x)$, where $x := |h_{k,2}^m|^2 p_{k,2}^m(\mathbf{h})$ is the signal-to-noise ratio (SNR) for SU $m$ on channel $k$ for the SISO (Single-Input Single-Output) case.

Once the variables are introduced, we formulate the constraints that these variables need to satisfy. To ensure that at most one user transmits into a given band $k$, we need

$$\sum\nolimits_m w_{k,2}^m(\mathbf{h}) \leq 1, \quad \forall k. \tag{1}$$

We also consider that the maximum average (long-term) power the $m$th SU can transmit is $\check{p}_2^m$; hence,

$$\mathbb{E}\left[\sum\nolimits_k w_{k,2}^m(\mathbf{h}) p_{k,2}^m(\mathbf{h})\right] \leq \check{p}_2^m, \quad \forall m. \tag{2}$$

Next, we formulate the constraints that limit the damage to PUs. Two constraints are considered: a) limits on the long-term interfering power and b) limits on the long-term rate (capacity) loss experienced by the PUs. To formulate a), let $i_{k,1}^m(\mathbf{h}, p_{k,2}^m) := |h_{k,1}^m|^2 p_{k,2}^m(\mathbf{h})$ denote the interference power caused by the $m$-th SU to the $k$-th PU, and let $\check{p}_{k,1}$ denote the maximum average interfering power the PU can tolerate. Moreover, recall that the $m$th SU transmits in the $k$th channel only if $w_{k,2}^m(\mathbf{h}) = 1$. Then, we need

$$\mathbb{E}\left[\sum\nolimits_m w_{k,2}^m(\mathbf{h}) i_{k,1}^m(\mathbf{h}, p_{k,2}^m)\right] \leq \check{p}_{k,1}, \quad \forall k. \tag{3}$$

To formulate b), we define the function $r_{k,1}(x) := \log_2\left(1 + \frac{\gamma_{k,1}}{1+x}\right)$, where $\gamma_{k,1}$ and $x$ stand for the normalized PU-to-PU SNR and the interfering power at the $k$th PU receiver, respectively. With $\check{\varepsilon}_k \in (0,1)$ being the maximum (relative) rate loss that the SUs can cause to the $k$th PU, we need the long-term PU-to-PU rate being greater than $\check{r}_{k,1} := (1-\check{\varepsilon}_k)\mathbb{E}[r_{k,1}(0)]$. Mathematically,

$$\mathbb{E}\left[r_{k,1}\left(\sum\nolimits_m w_{k,2}^m(\mathbf{h}) i_{k,1}^m(\mathbf{h}, p_{k,2}^m)\right)\right] \geq \check{r}_{k,1}, \quad \forall k. \tag{4}$$

Under all previous considerations, the optimal RA is obtained as the solution of the following sum-average rate maximization:

$$\max_{\{w_{k,2}^m(\mathbf{h}), p_{k,2}^m(\mathbf{h})\}} \sum\nolimits_{k,m} \mathbb{E}\left[w_{k,2}^m(\mathbf{h}) r_{k,2}^m(|h_{k,2}^m|^2 p_{k,2}^m(\mathbf{h}))\right] \tag{5a}$$

$$\text{s. to}: \quad w_{k,2}^m(\mathbf{h}) \in \{0,1\}, \; p_{k,2}^m(\mathbf{h}) \in \mathcal{P}_{k,2}^m, \; (1)-(4) \tag{5b}$$

where "s. to" stands for "subject to", and the dependence of the optimization variables on $\mathbf{h}$ (equivalently $\mathbf{h}[n]$) and $p_{k,2}^m$ has been made explicit.

The main challenge to find the optimal RA is that (5) is not a convex problem. The most challenging source of non-convexity is that constraint (4) is not convex with respect to $p_{k,2}^m$. (All other sources of convexity can be easily resolved; see, e.g., [4] for details on a related problem.) Two undesirable consequences associated with lack of convexity are: zero duality gap is not guaranteed, and development of computationally efficient numerical algorithms to find the solution is not guaranteed either. Remarkably, it holds for (5) that: it exhibits zero duality gap (the key is that the source of non-convexity is averaged across the channel distribution [3]), and the unconstrained Lagrangian can be optimized in polynomial time (the optimization can be separated across channels, power levels, and users).

Specifically, with $\pi^m$, $\theta_k$ and $\rho_k$ denoting the Lagrange multipliers associated with (2), (3) and (4), the optimal solution to (5) is

$$\begin{aligned} \varphi_k^m(p_{k,2}^m[n]) \;&:=\; r_{k,2}^m(|h_{k,2}^m[n]|^2 p_{k,2}^m[n]) - \pi^m p_{k,2}^m[n] \\ &\quad - \theta_k |h_{k,1}^m[n]|^2 p_{k,2}^m[n] + \rho_k r_{k,1}(|h_{k,1}^m[n]|^2 p_{k,2}^m[n]) \quad (6) \\ p_{k,2}^{m*}[n] \;&:=\; \arg\max_{p_{k,2}^m[n] \in \mathcal{P}_{k,2}^m} \varphi_k^m(p_{k,2}^m[n]) \quad (7) \\ w_{k,2}^{m*}[n] \;&:=\; \mathbb{1}_{\{m = \arg\max_{\bullet} \varphi_k^{\bullet}(p_{k,2}^{\bullet *}[n])\}}. \quad (8) \end{aligned}$$

In words, for each $n$, the following steps are implemented. S1) For each $k$ and $m$, all $L_P$ power levels are substituted into (6) and the best one is selected [cf. (7)]. S2) For a given $k$, the values of $p_{k,2}^{m*}[n]$ from step S1 are substituted into (8) and the best SU is selected. S3) Step S2 is run for all channels $k$. As a result, for each $n$, $L_P MK$ closed forms have to be evaluated, rendering the computational complexity polynomial. Hence, the joint optimization boils down to separate optimization of (6), which can be interpreted as a link quality indicator that takes into account the benefits for the SU and the costs for the PU. If the CSI is not perfect, the only change required is to average the terms in (6) over the (instantaneous) channel imperfections [5]. However, this will increase the complexity because a Montecarlo approach to estimate such averages would be required (a robust approach could be used to avoid a excessive number of computations). Finally, fact of the optimal solution being decomposable in the dual domain also holds true when the interference constraint is formulated as an upper bound on the long-term probability of interfering the PUs [4].

## III. Optimal Beamforming

In this section, we extend the previous results to a setup where SUs are equipped with several antennas and implement adaptive beamforming. For simplicity, we consider the case of Multiple-Input Single-Output (MISO) channels, although the schemes can be easily modified to be used in Multiple-Input Multiple-Output (MIMO) channels.

Consider that each SU transmitter has $R_T$ antennas and let $\mathbf{v}_{k,2}^m$ denote the complex valued $R_T \times 1$ beamforming vector that SU $m$ uses to transmit on channel $k$. Vector $\mathbf{v}_{k,2}^m$ is unitary and its entry $r$ represents the electromagnetic field transmitted by SU $m$ on channel $k$ through its $r$th antenna. To facilitate practical implementation, beamformers typically belong to a finite-size predesigned codebook [8], [7], [6]. Consequently, we consider that $\mathbf{v}_{k,2}^m \in \mathcal{V}_{k,2}^m$ where $\mathcal{V}_{k,2}^m := \{\mathbf{v}_{k,2}^{m,l}\}_{l=1}^{L_V}$ is the set of $L_V$ possible vectors.

In this new setup, the MISO channels between the SU transmitters and the SU and PU receivers are represented by complex vectors. Specifically, the MISO channel between SU $m$ and its intended receiver on channel $k$ is $\mathbf{h}_{k,2}^m := [h_{k,2}^{m,1}, \ldots, h_{k,2}^{m,R_T}]^T$, where $h_{k,2}^{m,r}$ denotes the complex channel coefficient between the $r$th transmitting antenna and the single receiver's antenna, and $^T$ denotes vector transposition. The MISO channel between SU transmitter $m$ and PU receiver $k$ is redefined analogously and it is denoted as the $R_T \times 1$ complex vector $\mathbf{h}_{k,1}^m$.

If the SU transmits with power $p_{k,2}^m$ and uses the beamforming vector $\mathbf{v}_{k,2}^m$, the SNR at SU receiver $m$ on channel $k$ is $p_{k,2}^m |\mathbf{v}_{k,2}^{mT} \mathbf{h}_{k,2}^m|^2$. Similarly, the interference power at PU receiver $k$ generated by SU transmitter $m$ is given by $i_{k,1}^m = p_{k,2}^m |\mathbf{v}_{k,2}^{mT} \mathbf{h}_{k,1}^m|^2$.

The next step is to update the constraints in (5) to accommodate the MISO vector channels and the adaptive beamforming. First, constraints in (1) and (2) do not require changes. Note, however, that the orthogonality constraint in (1) can give rise to a high loss of performance when transmitters are equipped with multiple antennas. Second, the constraint on the maximum interference power in (3) still needs to hold, but the new definition of $i_{k,1}^m$, which now depends on $\mathbf{v}_{k,2}^m$ and $\mathbf{h}_{k,2}^m$, has to be considered. Analogously, the constraint on the maximum capacity loss in (4) needs to account for the new definition of $i_{k,1}^m$.

Taking into account these considerations, the optimal RA for the MISO case is obtained as the solution of following sum-average rate maximization:

$$\max_{\substack{\{w_{k,2}^m(\mathbf{h}), p_{k,2}^m(\mathbf{h}),\\ \mathbf{v}_{k,2}^m(\mathbf{h})\}}} \sum\nolimits_{k,m} \mathbb{E}\left[ w_{k,2}^m(\mathbf{h}) r_{k,2}^m (p_{k,2}^m |\mathbf{v}_{k,2}^{mT} \mathbf{h}_{k,2}^m|^2)\right] \tag{9a}$$

$$\text{s. to} : w_{k,2}^m(\mathbf{h}) \in \{0,1\}, p_{k,2}^m \in \mathcal{P}_{k,2}^m, \mathbf{v}_{k,2}^m \in \mathcal{V}_{k,2}^m \quad (1),\ (2),\ (3),\ (4) \tag{9b}$$

The optimal solution to (9) can be found following steps similar to those in the previous section. The expressions for the optimal resource allocation can be found in (10a)-(10c), which are located at the top of the next page.

### A. Remarks on the codebook design

In this paper, we have focused on designing adaptive schemes for underlay CRs where the power and beamforming vectors (resources to be allocated) are chosen from the finite-size codebooks $\mathcal{P}_{k,2}^m$ and $\mathcal{V}_{k,2}^m$. Such codebooks can be different for each user and channel, have to be designed off-line (or during the initialization phase of the system) and are kept fixed during the communication phase. Regardless of the application, design of optimal quantizers is a difficult problem (typically NP hard), so that practical designs aim to obtain suboptimal solutions [9], [10]. There are different approaches to accomplish that [10]. A classical one is to find a local optimum of the original problem. The celebrated Lloyd algorithm is an example of such algorithms. Many works in the field of adaptive communications have developed modified versions of the Lloyd algorithm to design the power/beamforming codebooks. This approach could also be adopted for the problem at hand. However, the Lloyd algorithm is run in an iterative way, so that many iterations may be required until convergence occurs (specially when the original problem includes constraints and those are dualized [9]). An alternative approach is to use a criterion to design the quantizer different from the one optimized during the communication phase. Such criterion is selected to render the design of the quantizer tractable [10]. Although, the Lloyd algorithm typically gives rise to a slightly better performance, the decrease of the computational burden makes this second alternative very competitive. For the problem considered in this paper, reasonable choices to design the codebooks are: a) an equally-probable quantizer for the power levels in $\mathcal{P}_{k,2}^m$ [9], and a Grassmannian quantizer for the beamforming vectors in $\mathcal{V}_{k,2}^m$ [6]. The equally-probable quantizer would set the power levels so that the probability of selecting them during the communication phase is the same. Grossly speaking, the Grassmannian quantizer designs the beamformer codebook so that the set where the non-quantized beamformers belong to (complex unitary Grassmann manifold) is evenly sampled [6], [10]. Obviously, the performance of those quantizers should be compared with that of other alternatives. The design and subsequent performance analysis of the codebooks $\mathcal{P}_{k,2}^m$ and $\mathcal{V}_{k,2}^m$ for the setup considered in the paper is a problem of interest. However, it exceeds the scope of the manuscript and is left as future work.

## IV. Numerical simulations

In this section, we run several numerical tests to asses the performance of our algorithm (labeled as A1) and to compare it to that of: i) the optimal solution that uses perfect CSI [5] (labeled as A2); and ii) a suboptimal scheme (labeled as A3) that limits the interference using short-term constraints, which is a classical (widely-used) approach in the literature. First, in Test Case 1 we investigate how the three algorithms perform for different SNRs (i.e., varying $\mathbb{E}[|h_{k,1}^m|^2]$ and $\mathbb{E}[|h_{k,2}^m|^2]$). Second, to gauge if the findings in the Test Case 1 hold also for another scenarios, in Test Case 2 we modify the value of other parameters. The values for the default test case are summarized in Table I. The power codebook has been chosen to be a regularly spaced (in logarithmic units) set of values between a very small one and the maximum peak power that SUs are allowed to transmit. The beamforming codebook is designed using a Grassmannian quantizer [6] for $R_T = 3$ antennas and different codebook lengths.

*1) Test Case 1:* The purpose of this test is to compare algorithms A1, A2 and A3 using two basic configurations (namely, 6 bit-feedback and 4 bit-feedback) for different values

TABLE I: Parameters.

| Parameter | Value |
|---|---|
| M | 5 |
| K | 10 |
| $\check{p}_2^m$ | 1 Watt |
| $\gamma_{k,1}$ | 10dB |
| Fading distribution | Rayleigh |
| Number of paths | 6 |
| Coherence time | 300 symbols |
| $R_T$ | 3 |
| SU Max. Peak Power | 4 Watt |

$$\varphi_k^m(p_{k,2}^m[n], \mathbf{v}_{k,2}^m[n]) := r_{k,2}^m(p_{k,2}^m[n]|\mathbf{v}_{k,2}^{mT}[n]\mathbf{h}_{k,2}^m[n]|^2) - \pi^m p_{k,2}^m[n] - \theta_k p_{k,2}^m[n]|\mathbf{v}_{k,2}^{mT}[n]\mathbf{h}_{k,1}^m[n]|^2 + \rho_k r_{k,1}(p_{k,2}^m[n]|\mathbf{v}_{k,2}^{mT}[n]\mathbf{h}_{k,1}^m[n]|^2) \tag{10a}$$

$$(p_{k,2}^{m*}[n], \mathbf{v}_{k,2}^{m*}[n]) := \underset{p_{k,2}^m[n]\in\mathcal{P}_{k,2}^m, \mathbf{v}_{k,2}^m[n]\in\mathcal{V}_{k,2}^m}{\arg\max} \varphi_k^m(p_{k,2}^m[n], \mathbf{v}_{k,2}^m[n]) \tag{10b}$$

$$w_{k,2}^{m*}[n] := \mathbb{1}_{\{m=\arg\max_q \varphi_k^q(p_{k,2}^{q*}[n], \mathbf{v}_{k,2}^{q*}[n])\}} . \tag{10c}$$

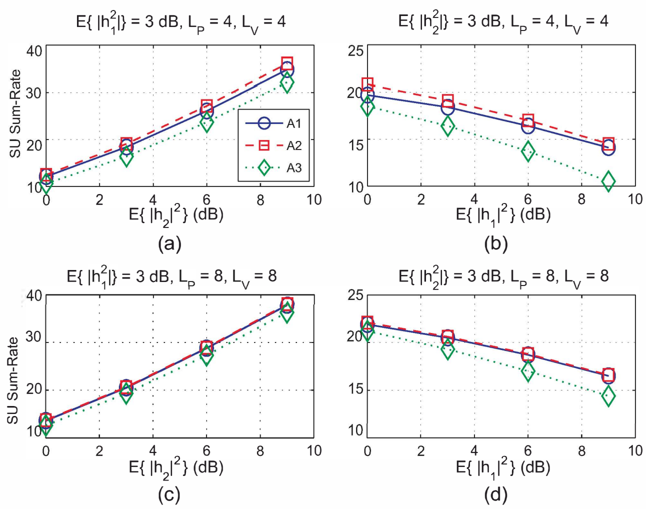


Fig. 1: SU sum rate achieved by the three algorithms for 4-bit feedback –(a) and (b)–, and for 6-bit feedback –(c) and (d)–. In (a) and (c) the results are given for different values of $\mathbb{E}[|h_{k,2}^m|^2]$. Analogously, in (b) and (d) the results are given for different values of $\mathbb{E}[|h_{k,1}^m|^2]$.

of the average SNR. First, we set the SU-to-PU average SNR ($\mathbb{E}[|h_{k,1}^m|^2] = 3$dB) and vary the SU-to-SU average SNR (see Fig. 1(a) and (c)). Second, we set the SU-to-SU average SNR ($\mathbb{E}[|h_{k,2}^m|^2] = 3$dB) and test several values for the SU-to-PU average SNR (see Fig. 1(b) and (d)). The upper plots in Fig. 1 represent the sum rate achieved by the SUs for $L_P = L_V = 4$ (4-bit feedback), and the lower plots for $L_P = L_V = 8$ (6-bit feedback). For these experiments, we set $\check{p}_{k,1} = 0.2$ and $\check{\varepsilon}_k = 0.1$. In all cases, the performance achieved by A1 and A2 is very similar (obviously, almost identical when A1 uses a high number of feedback bits), while the performance gap between A1 and A3 is larger for more challenging scenarios (i.e., lower SU-to-SU SNRs or higher SU-to-PU SNRs). Table II lists the average interference power and the average rate loss at the PUs. It can be observed that, in all cases, the constraints are satisfied. (Small violations are due to the fact that averages are found using a finite number of samples.)

*2) Test Case 2:* The objective of this test is to investigate the performance of A1 in different scenarios and using different codebook sizes. For each scenario, the average sum rate, interference power and capacity loss are provided. To analyze the effect of different power and beamforming codebooks sizes ($L_P$ and $L_V$), two different tests are run for each scenario.

TABLE II: Interference power and rate loss at the PUs for Test Case 1. $\mathbb{E}[h_{k,1}^m]$ and $\mathbb{E}[h_{k,2}^m]$ are expressed in dB.

| $\mathbb{E}[h_{k,1}^m] = 3$dB | | | | |
|---|---|---|---|---|
| | $L_P = 4$, $L_V = 4$ | | $L_P = 8$, $L_V = 8$ | |
| Alg / $\mathbb{E}[h_{k,2}^m]$ | Int. Power | Rate Loss | Int. Power | Rate Loss |
| A1 / 0 | 0.20 | 0.06 | 0.20 | 0.07 |
| A1 / 3 | 0.20 | 0.07 | 0.21 | 0.07 |
| A1 / 6 | 0.21 | 0.07 | 0.21 | 0.07 |
| A1 / 9 | 0.21 | 0.07 | 0.21 | 0.07 |
| A2 / 0 | 0.20 | 0.07 | 0.20 | 0.07 |
| A2 / 3 | 0.20 | 0.07 | 0.21 | 0.07 |
| A2 / 6 | 0.21 | 0.07 | 0.21 | 0.07 |
| A2 / 9 | 0.21 | 0.07 | 0.21 | 0.07 |
| A3 / 0 | 0.11 | 0.04 | 0.13 | 0.05 |
| A3 / 3 | 0.12 | 0.04 | 0.13 | 0.04 |
| A3 / 6 | 0.12 | 0.04 | 0.13 | 0.05 |
| A3 / 9 | 0.12 | 0.04 | 0.13 | 0.05 |
| $\mathbb{E}[h_{k,2}^m] = 3$dB | | | | |
| | $L_P = 4$, $L_V = 4$ | | $L_P = 8$, $L_V = 8$ | |
| Alg / $\mathbb{E}[h_{k,1}^m]$ | Int. Power | Rate Loss | Int. Power | Rate Loss |
| A1 / 0 | 0.19 | 0.06 | 0.20 | 0.06 |
| A1 / 3 | 0.20 | 0.07 | 0.21 | 0.07 |
| A1 / 6 | 0.21 | 0.07 | 0.21 | 0.07 |
| A1 / 9 | 0.21 | 0.07 | 0.21 | 0.07 |
| A2 / 0 | 0.19 | 0.06 | 0.20 | 0.06 |
| A2 / 3 | 0.20 | 0.07 | 0.21 | 0.07 |
| A2 / 6 | 0.21 | 0.07 | 0.21 | 0.07 |
| A2 / 9 | 0.21 | 0.07 | 0.21 | 0.07 |
| A3 / 0 | 0.10 | 0.03 | 0.12 | 0.04 |
| A3 / 3 | 0.12 | 0.04 | 0.13 | 0.04 |
| A3 / 6 | 0.13 | 0.04 | 0.14 | 0.05 |
| A3 / 9 | 0.13 | 0.04 | 0.15 | 0.05 |

First (left subplots in Fig. 2), we vary the value of $L_V$, but keep $L = L_P L_V$ constant (so that the number of feedback bits does not change). Second (right subplots in Fig. 2), several representative combinations of $L_P$ and $L_V$ are tested.

Regarding the configuration of the scenarios, we first examine the behavior of A1-A3 in a setup that is not very challenging, with $E[|h_1|^2] = 0$dB, $E[|h_2|^2] = 3$dB, $\check{p}_{k,1} = 0.3$ and $\check{\varepsilon}_k = 0.1$ (Fig. 2(a) and (b)). The results show that the three algorithms are able to satisfy interference and rate loss constraints. For A1 and A2 the power constraint is the one that is active (i.e., the one that is satisfied with equality), while A3 oversatisfies both constraints. Also, as in Test Case 1, the performance achieved by A1 is very similar to that achieved by A2. Note that A3 always provides a poorer sum-rate performance (because it oversatisfies the constraints). Second, we modify the average SNRs to simulate a more challenging (demanding) scenario. Specifically, we set $E[|h_1|^2] = 3$dB, $E[|h_2|^2] = 0$dB. The findings based on the results for this setup (plotted in Fig. 2(c) and (d)) are very similar to those in the previous one, although we observe that the difference between A1 and A3 increases slightly. Finally, using the SNRs of the first scenario, we change the maximum rate loss constraint at the PUs, setting $\breve{\epsilon}_k = 0.05$. The two main observations are: i) for A1 and A2, now the rate loss constraint is the one that is active; and ii) the performance gap between A3 and the other two schemes is larger. Both findings confirming that this is a more demanding scenario.

## V. Conclusions

We have designed stochastic schemes for CRs where secondary users transmit orthogonally and adapt their power loading and beamforming vector to optimize the performance of the secondary network while limiting the interference to primary users. Both the power level and the beamforming vector were selected from a finite-size codebook. The metric optimized was the average sum rate transmitted by the secondary users and the interference was quantified as the loss on the average capacity caused to the primary users. Limits on the average interference power were considered too. Although the formulated problem was non-convex, the *global* optimum solution was obtained using a dual decomposition approach. It turned out that finding the optimal resource allocation required the evaluation of $MKL_PL_V$ terms per slot, where $M$ is the number of secondary users, $K$ the number of primary users (one per channel), $L_P$ the size of the power codebook, and $L_V$ the size of the beamforming codebook. The design of the optimal codebooks was left as future work.

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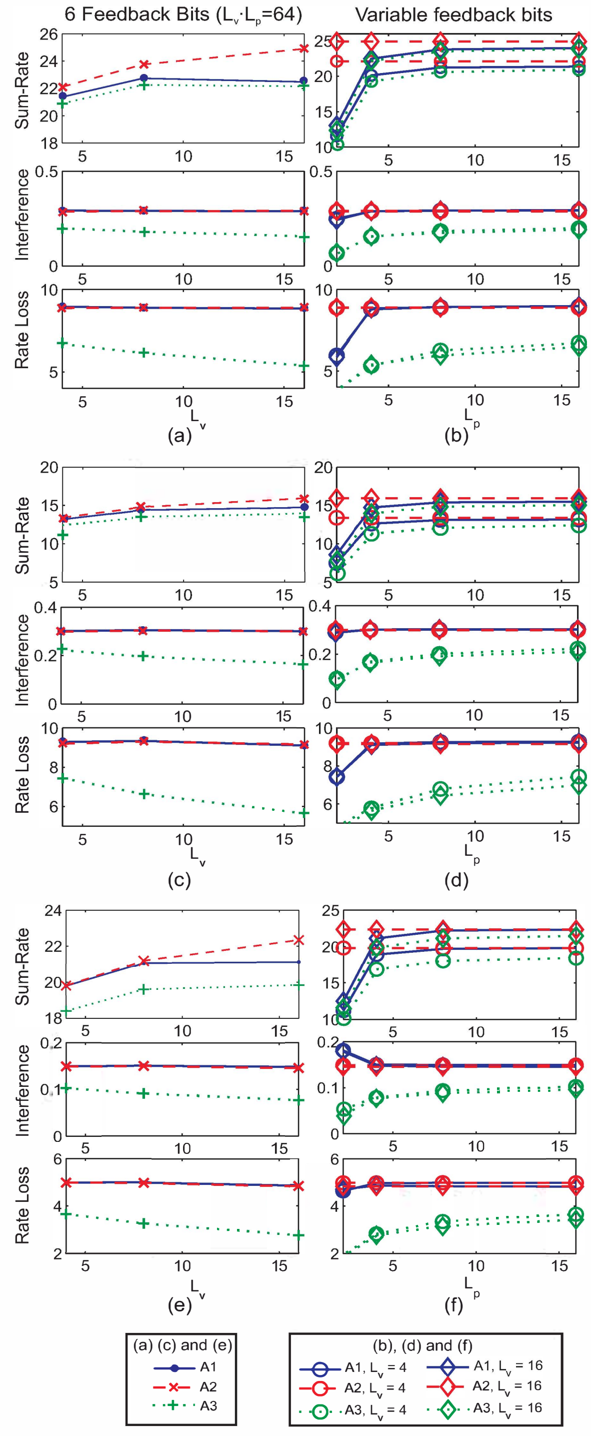


Fig. 2: SU sum-rate, mean interference power and mean rate loss at the PUs. Left subplots are obtained for fixed 6 bit-feedback. Right subplots are obtained for representative combinations of $L_p$ and $L_V$. Three scenarios are represented: $E[|h_1|^2] = 0$dB, $E[|h_2|^2] = 3$dB, $\breve{p}_{k,1} = 0.3$ and $\breve{\epsilon}_k = 0.1$, (a) and (b); $E[|h_1|^2] = 3$dB, $E[|h_2|^2] = 0$dB, $\breve{p}_{k,1} = 0.3$ and $\breve{\epsilon}_k = 0.1$, (c) and (d); and $E[|h_1|^2] = 0$dB, $E[|h_2|^2] = 3$dB, $\breve{p}_{k,1} = 0.3$ and $\breve{\epsilon}_k = 0.05$, (e) and (f).