

# STOCHASTIC RESOURCE ALLOCATION FOR COGNITIVE RADIO NETWORKS BASED ON IMPERFECT STATE INFORMATION

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## ABSTRACT

Efficient design of cognitive radio networks calls for secondary users implementing adaptive resource allocation, which requires knowledge of the channel state information in order to limit interference inflicted to primary users. In this context, the present paper develops stochastic resource allocation algorithms maximizing the sum-rate of secondary users while adhering to “average power” and “probability of interference” constraints. These constraints guarantee that the probability of the secondary network interfering with the primary one stays below a pre-specified level. The optimal schemes turn out to be a function of the quality of the secondary network links, the activity of the primary users, and the associated Lagrange multipliers. The focus is on algorithms that: i) use stochastic approximation tools to estimate the multipliers; and ii) are able to cope with imperfections in the information of the primary network state.

**Index Terms**— Cognitive radio, resource management, stochastic approximation, imperfect channel state information.

## 1. INTRODUCTION

The perceived spectrum under-utilization along with the proliferation of new wireless services have motivated recent research on dynamic spectrum management and cognitive radios (CRs), which are capable of sensing and accessing the spectrum dynamically. One of the most promising scenarios for deployment of CR networks consists of CR users (referred to as secondary users) communicating opportunistically in order to regulate interference caused to users holding licence of the frequency band (referred to as primary users) [4]. Secondary CRs need to intelligently sense the radio spectrum with two goals in mind: G1) Track the activity of primary users so that interference to the primary network is kept under control; and G2) Estimate the quality of the secondary network links so that fading can be mitigated and secondary users can take advantage of favorable channel opportunities. Based on measurements obtained through sensing, secondary users can thus adapt their available resources (here, power and rate) to the channel conditions, and decide whether to access or not a frequency band.

The merits of adaptive schemes which exploit knowledge of statistical and instantaneous channel *state information* (SI) to optimally allocate the transmit resources in traditional wireless systems are well documented; see e.g., [3, Chap. 9]. However, for channel-adaptive schemes to be employed by CRs, several challenges not present in traditional wireless networks need to be considered [8, 5,

9, 10, 11]. The most critical *design challenges* include: DC1) The need to satisfy additional constraints enforcing interference control; DC2) The fast variation of the CR environment which challenges availability of the statistical channel SI; and DC3) The heterogeneity and difficulty of acquiring instantaneous channel SI. To elaborate further on DC3, knowledge of the channel *SI of the secondary network* (SISN) is easier to acquire than that of the *SI of the primary network* (SIPN), since collaboration of the primary with the secondary users is typically limited (if any).

Different options are available to cope with DC1-DC3. To address DC1, some works limit the interference power at the primary receiver either by imposing instantaneous or average interference constraints (better suited for fading channels); see, e.g., [8, 5]. More recent designs limit the probability of interfering with primary transmissions [10, 11]. To deal with DC2, existing approaches resort to stochastic resource allocation; see [7, 11] for examples in the context of CRs. Several works have also pursued dual stochastic algorithms for resource allocation in wireless networks under different operating conditions [12, 6]. Such algorithms not only bypass the need to know the channel statistics and are robust to channel non-stationarities, but also decrease the computational complexity of the allocation schemes. Regarding DC3, different types of imperfections in the available SI have been considered. Pertinent works deal with either noisy SI [9], or quantized SI, where only a coarse description of the channel is available [7]). However, in the context of CR networks only a few works have considered that the SI may not only be noisy but also outdated [2].

Motivated by these considerations, low-complexity stochastic resource allocation algorithms are developed in this paper to optimize performance of the secondary network, limit its probability of interfering with the primary users, cope with channel non-stationarities, and account for outdated and noisy SIPN. The operating conditions assume that the available SISN is instantaneous and error free, while the available SIPN is outdated and noisy. A simple first-order hidden Markov model is adopted to characterize such imperfections, but the results also hold for more complex models. The secondary users access orthogonally a set of frequency bands (originally allocated to primary users) over which they adapt their power and rate loadings. Orthogonal here means that if a terminal is transmitting, no other link interfering with it can be active. The resource allocation schemes are then obtained as the solution of an optimization problem, which is formulated as a sum-average rate maximization subject to maximum “average power” and “probability of interference” constraints. The optimal resource allocation scheme turns out to be a function of the *instantaneous* SISN, the (possibly outdated and noisy) SIPN, and the optimum Lagrange multipliers estimated online using low-complexity stochastic iterations.

Work in this paper was supported by the Spanish Ministry of Science and Innovation grant No. TEC2009-12098; and the NSF grants CCF-0830480, 1016605, and ECCS-0824007, 1002180.

The operating conditions are presented in Section 2. The optimization problem is formulated in Section 3, and stochastic solvers under perfect SIPN are developed in Section 4. Imperfections in the SIPN are dealt with in Section 5. Numerical examples and conclusions wrap-up this paper.<sup>1</sup>

## 2. MODEL DESCRIPTION

Consider a CR network with  $M$  secondary users (indexed by  $m$ ) transmitting opportunistically over  $K$  frequency bands (indexed by  $k$ ). Suppose for simplicity that each band has identical bandwidth and is occupied by a different primary user. The secondary network has an access point (AP) responsible for sensing, scheduling, and resource allocation. It is assumed that at every instant, the instantaneous quality of the secondary links is available. Differently, occupancy of the frequency bands is not sensed at every time instant. These assumptions are well suited for scenarios where: i) sensing the state of the primary network entails a cost much higher than that of sensing the state of the secondary links; or ii) the primary network activity is bursty and therefore, there is no need for sensing it continuously. The channel's instantaneous power gain between the  $m$ th secondary user and the AP in the  $k$ th frequency band is denoted by  $h_k^m$ ; and represents the noise-normalized squared magnitude of the fading coefficient. Channels are assumed to be ergodic and generally correlated across bands. The primary network *activity* is represented by the boolean variable  $a_k$ , so that  $a_k = 1$  if the  $k$ th primary user is active and zero otherwise. Variable  $a_k$  is allowed to be correlated across time. Clearly, the value of  $h_k^m \forall (k, m)$  constitutes the SISN, while the value of  $a_k \forall k$  constitutes the SIPN. The overall SI is described by the random vector  $\mathbf{i}$  that collects all  $h_k^m$  and  $a_k$  values.

Regarding the design variables, let  $w_k^m$  denote a boolean variable so that  $w_k^m = 1$  if the  $m$ th user is scheduled to access the  $k$ th band, and  $w_k^m = 0$  otherwise. Provided that  $w_k^m = 1$ , let  $p_k^m$  denote the instantaneous power transmitted over band  $k$  by user  $m$ . Under bit error rate or capacity constraints, instantaneous rate and power variables are coupled. This rate-power coupling will be represented by the function  $C_k^m(h_k^m, p_k^m)$ . It is assumed throughout that the rate-power function  $C_k^m(h_k^m, \cdot)$  is given by Shannon's capacity formula  $\log(1 + h_k^m p_k^m / \Gamma_k^m)$ , where  $\Gamma_k^m$  represents the SNR-gap which depends on the coding scheme implemented [3].

The secondary CR network operates in a time-block fashion, where the duration of each block corresponds to the coherence time of the fading channel. At every time  $n$ , the AP relies on the current SI  $\mathbf{i}$  to optimize  $w_k^m$  and  $p_k^m$ . Since  $\mathbf{i}$  depends on  $n$  and  $\{w_k^m, p_k^m\}$  depend on  $\mathbf{i}$ , the design variables  $\{w_k^m, p_k^m\}$  will vary across time. Wherever convenient to stress the corresponding dependence, the notation  $w_k^m(\mathbf{i})$  and  $p_k^m(\mathbf{i})$ , or  $\mathbf{i}[n]$ ,  $w_k^m[n]$  and  $p_k^m[n]$ , will be used.

For this CR configuration, the goal is to develop adaptive algorithms that use the instantaneous SISN and the (possibly outdated and noisy) SIPN to optimally schedule the secondary users transmitting on each band, along with their rate and power loadings.

## 3. PROBLEM FORMULATION

The first step is to identify the constraints that the optimal schemes must satisfy. The power variables  $p_k^m$  are constrained to be non-

<sup>1</sup>Notation:  $x^*$  denotes the optimal value of variable  $x$ ;  $\wedge$  the boolean "and" operator;  $\mathbb{1}_{\{x\}}$  the indicator function ( $\mathbb{1}_{\{x\}} = 1$  if  $x$  is true and zero otherwise); and  $[x]_+$  the projection of  $x$  onto the non-negative orthant, i.e.,  $[x]_+ := \max\{x, 0\}$ . Finally, for a function  $f(\cdot)$ ,  $(f)^{-1}(\cdot)$  denotes its inverse and  $\dot{f}(\cdot)$  its derivative.

negative and the boolean variables  $w_k^m$  are constrained to belong to the set  $\{0, 1\}$ . Moreover, to ensure that at most one user can access a given band  $k$ , it is necessary to enforce the following scheduling constraint

$$\sum_k w_k^m(\mathbf{i}) \leq 1, \quad \forall k. \quad (1)$$

We also consider that the maximum average transmit-power of the  $m$ th secondary user is  $\bar{p}^m$ , which under the ergodic SI, means that

$$\mathbb{E} \left[ \sum_k w_k^m(\mathbf{i}) p_k^m(\mathbf{i}) \right] \leq \bar{p}^m, \quad \forall m \quad (2)$$

where expectations are taken over  $\mathbf{i}$ . Finally, control interference to the primary network, a maximum probability of interference  $\delta_k$  is allowed per band. Interference occurs when  $a_k = 1$  (primary user active in the  $k$ th band) and  $\sum_m w_k^m(\mathbf{i}) = 1$  (a secondary user is transmitting into the  $k$ th band). Hence, limiting the probability of interference amounts to bound  $\Pr\{\sum_m w_k^m(\mathbf{i}) = 1 | a_k = 1\} \leq \delta_k$ . Clearly, using Bayes' rule and the fact that  $w_k^m$  and  $a_k$  are boolean variables, the previous inequality can be alternatively written as

$$\mathbb{E} \left[ a_k \sum_m w_k^m(\mathbf{i}) \right] / A_k \leq \delta_k, \quad \forall k, \quad (3)$$

where  $A_k$  denotes the probability of the  $k$ th band being occupied by the corresponding primary user.

The second step is to define the proper metric to be optimized. This is the sum-average rate  $\bar{c} := \sum_k \mathbb{E} [w_k^m(\mathbf{i}) C_k^m(h_k^m, p_k^m(\mathbf{i}))]$ , although other objective functions such as weighted sum-rate or sum-utility can be used without changing the basic structure of the solution; see, e.g., [6] for further details.

Under all previous considerations, the optimal resource allocation scheme is obtained as the solution of the following problem:

$$\bar{c}^* := \max_{w_k^m(\mathbf{i}), p_k^m(\mathbf{i})} \sum_k \mathbb{E} [w_k^m(\mathbf{i}) C_k^m(h_k^m, p_k^m(\mathbf{i}))] \quad (4a)$$

$$\text{s. to: (1), (2), (3), } w_k^m(\mathbf{i}) \in \{0, 1\}, \text{ and } p_k^m(\mathbf{i}) \geq 0 \quad (4b)$$

where the dependence of the optimization variables on the SI  $\mathbf{i}$  has been made explicit.

## 4. SOLUTION WITH PERFECT SIPN

### 4.1. Optimum solution without interference constraints

To gain insight, consider first solving (4) without the constraint in (3). Although the problem in (4) is not convex, it can be trivially transformed (relaxed) to a convex one, which yields the same Karush-Kuhn-Tucker (KKT) conditions<sup>2</sup>. Without constraint (3), the problem in (4) reduces to a sum-rate optimization of an uplink channel with orthogonal access. With  $\pi^m$  denoting the Lagrange multiplier

<sup>2</sup>There are two sources of non-convexity in (4). The first one is the non-convexity of the set  $w_k^m \in \{0, 1\}$ . This can be solved by relaxing that set and allow  $w_k^m \in [0, 1]$ . Since  $w_k^m$  is a random variable that only appears in linear terms, it can be shown that this relaxed solution coincides with the original one with probability one. The second source of non-convexity is the presence of the monomials  $w_k^m p_k^m$  and  $w_k^m C_k^m$ . This can be trivially addressed by introducing dummy variables  $\tilde{p}_k^m := w_k^m p_k^m$  in (4) and showing convexity using the properties of the perspective function. The resulting convex problem yields the same KKT conditions that (4), and can be solved using a dual approach. As in the remainder of this paper, proofs are omitted due to space limitations, see e.g., [13] for details.

associated with the constraint in (2), it has been shown that the solution is (see e.g., [13, 6])

$$p_k^{m*}[n] := \left[ (\dot{C}_k^m)^{-1} (h_k^m, \pi^m[n]) \right]_0^\infty; \quad (5)$$

$$w_k^{m*}[n] := \mathbb{1}_{\{(\varphi_k^m[n]=\max_l \varphi_k^l[n]) \wedge (\varphi_k^m[n]>0)\}}, \text{ with} \quad (6)$$

$$\varphi_k^m[n] := C_k^m(h_k^m[n], p_k^{m*}[n]) - \pi^m[n] p_k^{m*}[n]. \quad (7)$$

Note that: i) the closed form in (5) corresponds to the well-known waterfilling solution [3]; and ii) equations (6)-(7) corroborate that the user scheduling is opportunistic. Specifically, (7) can be readily interpreted as a user quality indicator (the higher the rate and the lower the power, the better); while (6) dictates that only the user with highest quality must be scheduled per band.

Different methods can be used to select  $\pi^m[n]$ . Traditionally,  $\pi^m[n]$  is set to a constant  $\pi^{m*}$  corresponding to the value maximizing the dual function associated with (4). Since the relaxed problem in (4) is convex and strictly feasible, the duality gap is zero [1]. This implies that if  $\pi^m[n] = \pi^{m*}$  is substituted into (5)-(7), the resulting resource allocation solves optimally (4). The main limitations of this approach are that: i)  $\pi^{m*}$  needs to be found through numerical search which, at every step, requires averaging over all possible states of  $\mathbf{i}$ ; and ii) if the number of users or the channel statistics change,  $\pi^{m*}$  must be recomputed. Alternative approaches that rely on stochastic approximation tools are available for estimating the multipliers [12, 6]. These approaches do not aim at the optimal value of  $\pi^{m*}$ , but an estimate of it which is updated at every time instant and remains sufficiently close to  $\pi^{m*}$ . The main advantages of these approaches are: i) low complexity; ii) ability to track channel non-stationarity; and iii) no need to know the underlying channel statistics. The price paid is that the resulting schemes are slightly suboptimal. Specifically, for the problem at hand the following iteration can be used to update the value of  $\pi^m[n]$

$$\pi^m[n+1] = \left[ \pi^m[n] - \mu(\check{p}^m - \sum_k w_k^{m*}[n] p_k^{m*}[n]) \right]_+, \quad (8)$$

where  $\mu$  denotes a small stepsize. The update in (8) is an unbiased stochastic version of the dual function subgradient in (4); see [6, 1].

If the updates in (8) are bounded, it can be shown that the sample average of the stochastic resource allocation: i) is feasible, and ii) entails a small loss in performance relative to the optimal solution of (4). Concretely stated, if  $\check{p}^m[n] := \frac{1}{n} \sum_{l=1}^n \sum_k w_k^{m*}[l] p_k^{m*}[l]$ , and  $\check{c}[n] := \frac{1}{n} \sum_{l=1}^n \sum_{k,m} w_k^{m*}[l] C_k^m(h_k^m[l], p_k^{m*}[l])$ , it then holds with probability one that as  $n \rightarrow \infty$ : i)  $\check{p}^m[n] = \check{p}^m$ ; and, ii)  $\check{c}[n] \geq \check{c}^* - \delta(\mu)$ , where  $\delta(\mu) \rightarrow 0$  as  $\mu \rightarrow 0$ .

#### 4.2. Optimum solution with interference constraints

Consider now the original formulation in (4). Letting  $\theta_k$  denote the Lagrange multiplier associated with the constraint in (3), the new optimal allocation is the following

$$p_k^{m*}[n] := \left[ (\dot{C}_k^m)^{-1} (h_k^m, \pi^m[n]) \right]_0^\infty; \quad (9)$$

$$w_k^{m*}[n] := \mathbb{1}_{\{(\varphi_k^m[n]=\max_l \varphi_k^l[n]) \wedge (\varphi_k^m[n]>0)\}}, \text{ with} \quad (10)$$

$$\varphi_k^m[n] := \varphi_k^m[n] - \theta_k[n] a_k[n]. \quad (11)$$

The only difference between (5)-(7) and (9)-(11) is the redefinition of the quality indicator in (11). On top of considering the trade-off between rate and power, (11) also penalizes those secondary transmissions that cause interference to the primary user. Since  $\theta_k[n] a_k[n]$

does not depend on  $m$ , the winner user in (6) and (10) is the same. The difference though is that if  $\theta_k[n] a_k[n] \geq \max_m \varphi_k^m[n]$ , then  $\varphi_k^m[n] \leq 0 \forall m$ , and no user will transmit on the  $k$ th band. In other words, when  $a_k[n] = 1$ , the secondary network will transmit only if the quality of the secondary link is so high that exceeds the cost of interference quantified by  $\theta_k[n]$ .

As in the case of  $\pi^m[n]$ , there are several possibilities in selecting  $\theta_k[n]$ . Following the stochastic approach of the previous section, the following online update is proposed

$$\theta_k[n+1] = \left[ \theta_k[n] - \mu(\check{\theta}_k - (a_k[n]/A_k) \sum_m w_k^{m*}[n]) \right]_+. \quad (12)$$

The convergence, feasibility, and optimality properties of iteration (12) are similar to those stated after (8).

### 5. SOLUTION WITH IMPERFECT SIPN

So far, the SIPN available was assumed perfect. However, SIPN will typically contain errors, which implies that at every time instant  $n$ , the exact  $a_k[n]$  will be unknown. Only the belief state of  $a_k[n]$ , defined as  $\hat{\mathbf{a}}_k[n] := [\Pr\{a_k[n] = 0\}, \Pr\{a_k[n] = 1\}]^T$  will be available. Note that  $\hat{\mathbf{a}}_k[n]$  can be understood as the probability mass of  $a_k[n]$  based on the system history up to the instant  $n$ . To illustrate the importance of SIPN uncertainty in the performance of the optimal schemes, consider the following example. Suppose that the  $k$ th primary user is active during  $A_k = 40\%$  of the time, and that the allowed interference is set to  $\check{\theta}_k = 5\%$ . For the extreme case where SIPN measurements are totally erroneous, the secondary network can use the  $k$ th band during 5% of the time. In contrast, if the SIPN measurements are error free so that  $\hat{\mathbf{a}}_k[n] = [(1 - a_k[n]), a_k[n]]^T$ , the secondary network can use the  $k$ th band during  $60\% + 5\% \times 40\% = 62\%$  of the time.

#### 5.1. Modeling the SIPN imperfections

Let  $s_k[n]$  denote a boolean variable, which equals one if the  $k$ th band is sensed at instant  $n$ , and zero otherwise. Moreover, let  $\tilde{a}_k[n]$  denote the (possible noisy) measurement of  $a_k[n]$  obtained at instant  $n$  if  $s_k[n] = 1$ . Two are the main sources of SIPN imperfections: i) outdated SIPN (for the instants  $n$  such that  $s_k[n] = 0$ ); and ii) noisy SIPN (due to errors in the sensing process that may render  $a_k[n] \neq \tilde{a}_k[n]$ ).

Consider modeling first the time dynamics of  $a_k[n]$  which, for simplicity, are assumed to follow a first-order Markov process. With  $P_{ji}$  denoting the probability  $\Pr\{a_k[n] = i | a_k[n-1] = j\}$ , define the transition probability matrix  $\mathbf{P} := [P_{00}, P_{10}; P_{01}, P_{11}]$ . To incorporate sensing errors into the model, consider also the probabilities of miss detection and false alarms given, respectively, by  $P_{MD} := \Pr\{\tilde{a}_k[n] = 0 | a_k[n] = 1\}$ , and  $P_{FA} := \Pr\{\tilde{a}_k[n] = 1 | a_k[n] = 0\}$ .

Based on this model, updating the belief state can proceed in the following cases:

- If  $s_k[n] = 0$ , then  $\hat{\mathbf{a}}_k[n] = \mathbf{P}\hat{\mathbf{a}}_k[n-1]$ .
- If  $s_k[n] = 1$  and  $\tilde{a}_k[n] = 0$ , update first the belief state of the previous instant to find  $\tilde{\mathbf{a}}_k[n] := \mathbf{P}\hat{\mathbf{a}}_k[n-1]$ ; and use that  $\tilde{a}_k[n] = 0$  to correct the prediction based on Bayes' rule:

$$[\hat{\mathbf{a}}_k[n]]_1 = \frac{(1 - P_{FA})[\tilde{\mathbf{a}}_k[n]]_1}{(1 - P_{FA})[\tilde{\mathbf{a}}_k[n]]_1 + P_{MD}[\tilde{\mathbf{a}}_k[n]]_2}$$

$$[\hat{\mathbf{a}}_k[n]]_2 = \frac{P_{MD}[\tilde{\mathbf{a}}_k[n]]_2}{(1 - P_{FA})[\tilde{\mathbf{a}}_k[n]]_1 + P_{MD}[\tilde{\mathbf{a}}_k[n]]_2},$$

where  $[\cdot]_l$  stands for the  $l$ th entry of a vector.

**Table 1.** Simulation results

	S1	S2	S3	S4	S5
$\bar{c}$	19.8	19.7	2.0	27.5	19.9
$(1/K) \sum_k \bar{o}_k$	5%	7%	5%	39.8%	5%

- If  $s_k[n] = 1$  and  $\tilde{a}_k[n] = 1$ , find first  $\tilde{\mathbf{a}}_k[n] := \mathbf{P}\hat{\mathbf{a}}_k[n-1]$ , and then correct the prediction update

$$[\hat{\mathbf{a}}_k[n]]_1 = \frac{P_{FA}[\tilde{\mathbf{a}}_k[n]]_1}{P_{FA}[\tilde{\mathbf{a}}_k[n]]_1 + (1 - P_{MD})[\tilde{\mathbf{a}}_k[n]]_2}$$

$$[\hat{\mathbf{a}}_k[n]]_2 = \frac{(1 - P_{MD})[\tilde{\mathbf{a}}_k[n]]_2}{P_{FA}[\tilde{\mathbf{a}}_k[n]]_1 + (1 - P_{MD})[\tilde{\mathbf{a}}_k[n]]_2}.$$

Note that the described procedure resembles the prediction-correction steps of a Kalman filter (only prediction if  $s_k[n] = 0$ , and both prediction and correction when  $s_k[n] = 1$ ).

## 5.2. Effect on the resource allocation

Imperfections in the SIPN will affect both the quality indicator in (11), as well as the stochastic update in (12).

- In the case of (11), the variable  $a_k[n]$  needs to be replaced with  $[\hat{\mathbf{a}}_k[n]]_2$ .
- In the case of (12), it suffices to replace  $a_k[n]$  with an unbiased estimate of it. Several options are possible, although for simplicity we choose  $[\hat{\mathbf{a}}_k[n]]_2$  also in this case.

## 6. NUMERICAL SIMULATIONS

The simulation setup is the following:  $M = 10$ ,  $K = 5$ ,  $\tilde{p}^m = 2$ ,  $\tilde{o}_k = 5\%$ ,  $A_k = 40\%$ , and  $\Gamma_k^m = 1$ . The amplitude of the secondary links is Rayleigh distributed, and the average SNR for all users is 0dB. The transition probabilities are  $P_{00} = 0.95$ ,  $P_{01} = 0.075$ ,  $P_{10} = 0.05$  and  $P_{11} = 0.925$ , and the errors in the sensing are  $P_{FA} = 5\%$  and  $P_{MD} = 3\%$ . The AP senses every 8 slots.

Table 1 lists the values of the sum-rate and average interference probability for the following schemes: S1) the optimal scheme of Section 5.2; S2) a scheme using error-free SIPN, which assumes that  $a_k[n + n_0] = \tilde{a}_k[n]$  for  $n_0 = 0, \dots, 7$ , and then implements the schemes of Section 4.2; S3) a scheme that ignores the SIPN and transmits only 5% of the time at instants corresponding to the best channel realizations of the secondary network; S4) a scheme that ignores the interference constraint and maximizes the sum-rate of the secondary network (as the one in Section 4.1); and S5) a genie-aided scheme that knows the actual value of the SIPN. The space limitations prevent us from a detailed discussion of the results. Nevertheless, the preliminary results suggest the theoretical claims are valid and illustrate the advantages of the developed algorithms.

## 7. CONCLUSIONS

Stochastic resource allocation algorithms were developed for secondary cognitive radios operating over wireless fading channels. The schemes were obtained by solving a sum-rate maximization problem subject to maximum “average power” and “probability of interference” constraints. The latter guarantee that the probability of the secondary network interfering with the primary one remains below

a prescribed level. It turned out that the optimal schemes were expressible in terms of the secondary network links; the activity of the primary users; and several Lagrange multipliers. The value of those multipliers depends on the system history, and the quality of service requirements of the primary and secondary networks. Stochastic algorithms that acquire the channel statistics on-the-fly, entail low computational complexity and have provable convergence, were proposed to estimate the multipliers. Imperfections in the state information of the primary network were also modeled to gauge their impact on the novel resource allocation schemes.

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