Power-Efficient OFDM via Quantized Channel State Information

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Abstract—In response to the growing demand for low-cost low-power wireless sensor networks and related applications, we develop bit and power loading algorithms that minimize transmit-power for orthogonal frequency division multiplexing (OFDM) under rate and error probability constraints. Our novel algorithms exploit one of three types of channel state information at the transmitter (CSIT): deterministic (per channel realization) for slow fading links, statistical (channel mean) for fast fading links, and quantized (Q-) CSIT whereby a limited number of bits are fed back from the transmitter to the receiver. By adopting average transmit-power as a distortion metric, a channel quantizer is also designed to obtain a suitable form of Q-CSI. Numerical examples corroborate the analytical claims and reveal that significant power savings result even with a few bits of Q-CSI.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been known for its spectral efficiency, error resilience, and ease in implementation. For these merits, OFDM has been adopted in many applications such as digital subscriber line (DSL), digital audio and video broadcasting (DAB/DVB) standards and wireless local area networks, to name a few [7], [13]. Unlike maximum-rate or minimum bit error rate (BER) driven applications (see e.g., [4], [8] and references therein), this paper is tailored for minimum power applications that have been recently receiving much attention in both commercial and tactical communication systems. Towards this objective, we rely on channel state information at transmitter (CSIT), to design optimum adaptive power and bit loading schemes that minimize transmit-power for OFDM under prescribed data rate and BER. This is in contrast to spectrally-efficient OFDM designs which maximize rate for a prescribed transmit power.

Three types of CSIT will be considered. The first is deterministic CSIT (D-CSIT) that has been used in DSL and can also be available the wireless channel is fading slowly. However, D-CSIT may be unrealistic to assume available in many wireless scenarios due to channel estimation errors at the receiver, feedback delay and asymmetry between forward and reverse links. This has motivated OFDM loading schemes based on statistical (S) or quantized (Q) CSIT, whereby a limited number of bits are fed back from the transmitter to the receiver; see e.g., [8], [14] and references therein.

Interestingly, except for [11] where D-CSIT is used to minimize power consumption, OFDM has not been optimized in the power-limited regime. It is this paper’s main goal to design power-efficient OFDM transceivers for frequency selective channels using Q-CSIT. We further consider the two limiting scenarios of either slow or fast fading for which we design power efficient OFDM schemes based on D-CSIT or S-CSIT. Our Q-CSIT based scheme will turn out to be much more power-efficient than the S-CSIT based one, and for a sufficient number of feedback bits it approaches the power savings achieved by the benchmark D-CSIT design.

The rest of the paper is organized as follows. Section II describes the setup. Power-efficient OFDM designs using D-CSIT, Q-CSIT, or S-CSIT are studied in Sections III, IV, and V, respectively. Finally, numerical results are provided in Section VI.

II. SYSTEM OVERVIEW

We consider a point-to-point wireless OFDM transmission using $N$ sub-carriers (sub-channels) through a frequency-selective channel with discrete-time baseband equivalent impulse response taps $\{h_k\}_{k=0}^{L}$, where $L := [D_{\text{max}} W]$ denotes the channel order, $D_{\text{max}}$ its maximum delay spread, $W$ the transmit-bandwidth, and $[\cdot]$ stands for the floor operation; see also Figure 1. A serial stream of data bits is first demultiplexed to form $N$ parallel streams each of which (indexed by $k \in [0, N-1]$) is multiplied by an appropriate constant to load power $P_k$, and modulated to obtain symbols $\{A_k\}_{k=0}^{N-1}$ drawn from a constellation of size $M_k$. The $N$-point inverse fast Fourier transform (I-FFT) applied to each snapshot of the $N$ symbol streams is followed by insertion of a size-$L$ cyclic prefix (CP) to yield a block of $N+L$ symbols (a.k.a. OFDM symbol), which are subsequently multiplexed...
channel gain vector $g_k := [g_0, \ldots, g_{N-1}]^T$, where $[\cdot]^T$ denotes transposition and $g_k := |H_k|^2/\sigma_k^2$ is the instantaneous noise-normalized channel gain of the $k$th sub-channel on which the zero-mean additive white Gaussian noise (AWGN) has variance $\sigma_k^2$. While each deterministic realization of the random gain $g_k$ constitutes the D-CSI, its statistical average $\bar{g}_k := \mathbb{E}[g_k]$ is what we henceforth refer to as S-CSI of the $k$th sub-channel. Either one must be available to the transmitter in the D-CSI or S-CSI based OFDM designs. For Q-CSI based design, sole knowledge of $\bar{g}_k$ specifies the implemented channel quantizer whereas both $g_k$ and quantized $\bar{g}_k$ determine bit and power allocation. The quantizer design entails selection of quantization thresholds $\tau_{k,j}$, which in turn define quantization regions $\mathcal{R}_{k,j}$ each sub-channel realization $g_k$ falls into. The random vector of bits $j := [j_0, j_1, \ldots, j_{N-1}]^T$ representing indices of the regions of all sub-channels is what we term Q-CSI.

Given either $g$ (D-CSI), or its mean $\bar{g}$ (S-CSI), or the quantized form of the channel gains specified by the bit vector $j$ (Q-CSI), our ultimate goal is to choose power and bit vectors $p := [P_0, \ldots, P_{N-1}]^T$ and $m := [M_0, \ldots, M_{N-1}]^T$ so as to minimize the power $P := \sum_{k=0}^{N-1} P_k$, or its average $\bar{P}$, for prescribed values of the rate (R0) and the bit error rate (BER0) per OFDM symbol $a := [A_0, \ldots, A_{N-1}]$. In addition, we wish to design a proper channel quantizer used by the receiver to provide the Q-CSI to the transmitter through the bit vector $j$. We will pursue these objectives under the following assumptions:

**a1.** Symbols $A_k$ are drawn from quadrature amplitude modulation (QAM) constellations of size $M_k$;

**a2.** Sub-channels $H_k$ remain invariant over at least two consecutive OFDM symbols, they are allowed to be correlated, and each is complex Gaussian distributed; i.e., each sub-channel gain $g_k$ adheres to an exponential probability density function (pdf)

$$f_{g_k}(g_k) = \left(1/\bar{g}_k\right) \exp(-g_k/\bar{g}_k);$$

**a3.** Feedback is error-free and incurs negligible delay.

Assumptions a1-a3 are common to existing designs [7], [4], [8], and are typically satisfied in practice. In principle, our results apply to any channel pdf, but a2 simplifies the resultant designs. Channel invariance in a2 allows for feedback delays; it may be stringent for D-CSIT based designs, but is very reasonable in the Q-CSIT case since $\bar{g}$ is allowed to vary from one OFDM symbol to the next so long as $j$ remains invariant. Finally, error-free feedback is easily guaranteed with sufficiently strong error control codes especially since rate in the reverse link is low.

With $R$ denoting rate (number of bits per OFDM symbol), the instantaneous BER is

$$\text{BER} = \frac{1}{R} \sum_{k=0}^{N-1} \log_2(M_k) \text{BER}_k(g_k, P_k, M_k)$$

where for the QAM constellations in a1, we have

$$\text{BER}_k(g_k, P_k, M_k) \simeq a \exp(-\beta_k g_k P_k)$$

and $a = 0.2$, $\beta_k := \frac{g_k}{M_k-1}$, with $b_k = 1$ if $M_k = 2$, or, $b_k = 1.5$ if $M_k \geq 4$. The approximation of the BER corresponding to the $k$th sub-channel is very accurate [3], [5].

**III. Power Optimization based on D-CSIT**

In this section, capitalizing on the D-CSIT gain vector $g$, we optimize power efficiency of OFDM for slow fading wireless or wireline channels given constraints on data rate and BER. In addition, the resultant performance based on the full CSI g serves as a benchmark for other designs that exploit partial (namely quantized or statistical) CSI. With $\mathcal{M}$ denoting the finite set of possible values of $m$, our objective can be formulated as:

$$\min_{m \in \mathcal{M}, p(g) \geq 0} J(p), \quad J(p) := \sum_{k=0}^{N-1} P_k(g_k)$$

s. t. $C1. \sum_{k=0}^{N-1} \frac{\log_2(M_k)}{R_0} \text{BER}_k(g_k, P_k, M_k) = \text{BER}_0$

$$C2. \sum_{k=0}^{N-1} \log_2 M_k = R_0.$$ 

Since entries in $p$ take on continuous values while those in $m$ are integer-valued, we deal with a mixed-integer programming (MIP) problem, which can be solved in two steps. First, for a fixed $m$, we remove $C2$ and thus solve (5) to obtain an optimum $p$ expressed in terms of $m$. Then, we test all feasible entries of $m$ that satisfy $C2$ to eventually select the overall optimum $(p, m)$ pair. Substituting (4) into (5), the first step is:

$$\min_{p(g) \geq 0} J(p), \quad J(p) := \sum_{k=0}^{N-1} P_k(g_k)$$

s. t. $C1. \sum_{k=0}^{N-1} \frac{a \log_2(M_k)}{R_0} e^{-\beta_k g_k P_k(g_k)} = \text{BER}_0.$$

Invoking the necessary Karush-Kuhn-Tucker (KKT) conditions for optimality [6, Sec. 14.6], the solution $P_k(g_k)$ can be found to be:

$$P_k(g_k) = \frac{1}{\beta_k g_k} \left[ \ln \left( \frac{a \beta_k g_k \log_2(M_k)}{R_0} \lambda_D \right) \right]^+,$$

where $[x]^+ := \max(x, 0)$, and $\lambda_D$ is the so-called Lagrange multiplier which can be obtained after plugging (7) into C1. Due to the convexity of both $J(p)$ and $C1$ over $p$, the solution in (7) is globally optimum. Reminiscent of the water-filling principle, (7) also shows that depending on sub-channel gains, it is possible for certain sub-channels to receive zero-power. Let $N_k$ be the set of indices corresponding to the $N_k \leq N$ active sub-channels which are allocated non-zero power. Upon substituting (7) into C1, we obtain $\lambda_D = \max \left\{ \sum_{k \in N_k} \frac{1}{\beta_k g_k} \right\}.$
We next incorporate the rate constraint $C_2$ in the second step of our optimization process, and reduce the search over $m$ considerably by sorting the channel gains in descending order and then assigning $M_{k_1} \geq M_{k_2}$ if $g_{k_1} \geq g_{k_2}$; see also [11]. The rationale behind this rule is that the higher $g_k$ is, the higher $M_k$ can be loaded to minimize $P$.

**Remark 1:** In the optimal loading schemes of this section as well as those of the ensuing sections we do not impose peak-to-average-power-ratio (PAPR) constraints. However, available digital pre-distortion schemes (such as the selected mapping algorithm in [12]) can be applied to each block of $\{A_k\}_{k=0}^{N-1}$ symbols in order to meet pre-specified PAPR constraints.

In the next section, we develop a power efficient OFDM scheme using Q-CSIT; i.e., only a few feedback bits.

### IV. Optimization Based on Q-CSIT

Unlike optimizing power efficiency of OFDM based on the realization $g$ carried out in the preceding section, here we fulfill the same objective while relying on the bit vector $j$ comprising codewords $j_k$ which represent the indices of the quantization regions each sub-channel gain $g_k$ falls into. In this context, we will assume that:

**a4. Codeword $j_k$ has length $B \forall k$; hence, $j$ is an $NB \times 1$ vector with 0, 1 entries.**

These $B$ bits per sub-channel will index $2^B$ regions $R_{k,j} := \{T_{k,j}, \tau_{k,j+1}\}$, that the $g_k$ space is divided into using quantization thresholds $\{\tau_{k,1}, \ldots, \tau_{k,2^B-1}\} := \tau_k$ for each sub-channel $k \in \{0, \ldots, N-1\}$. Notice that for brevity, we use $j$ (rather than $j_k$) to index regions and thresholds; however, since the latter include also the subscript $k$, we should keep in mind that regions and thresholds are allowed to be different from one sub-channel to another. Further, since quantization here pertains to gains, we set $\tau_{k,0} = 0$ and $\tau_{k,2^B} = \infty$.

Our Q-CSIT based optimization will proceed in two phases: a) specification of the optimum thresholds $\{\tau_k\}_{k=0}^{N-1}$, which can be performed off-line; and b) derivation of the optimum power and bit loading vectors $(m, p)$ to be used online based on the Q-CSI codeword vector $j$ obtained in accordance with the quantizer design phase a). We should keep in mind that after phase a) both transmitter and receiver need to know the optimal thresholds since they will use them in phase b).

#### A. Off-line Phase: Quantizer Design

Our goal here is to derive quantization thresholds for the random variable $g_k$ whose pdf is known as per a2. Those are obtained upon minimization of the underlying distortion metric which will be taken to be the average power $\sum_{k=0}^{N-1} P_k$. Therefore, the thresholds shall guarantee power savings across different channel realizations. We first note that

$$\tilde{P}_k = \sum_{j=0}^{2^B-1} P_{k,j} \Pr(P_k = P_{k,j}),$$

where $P_k$ is a discrete random variable taking values $\{P_{k,j}\}_{j=0}^{2^B-1}$ each denoting the power in the $k$th sub-channel and $j$th region having probability

$$\Pr(P_k = P_{k,j}) = \Pr(g_k \in R_{k,j}) := \int_{\tau_{k,j}}^{\tau_{k,j+1}} f_{g_k}(g_k) dg_k$$

$$= \exp(-\tau_{k,j}/\bar{g}_k) - \exp(-\tau_{k,j+1}/\bar{g}_k),$$

where the last equality results from the exponential pdf in a2. This in turn reflects the dependence of the average power on the quantizing thresholds. Along with minimizing the average power, the optimal thresholds should fulfill prescribed rate and average BER constraints. Expressing the average BER per sub-channel as

$$\text{BER}_k(\tau_k, \{P_{k,j}\}, M_k) = \sum_{j=0}^{2^B-1} \int_{\tau_{k,j}}^{\tau_{k,j+1}} \text{BER}_k(g_k, P_{k,j}, M_k) \times f_{g_k}(g_k) dg_k$$

(10)

(while for $j = 0$, we have $P_{k,0} = \text{BER}_k = 0$), our channel quantizer design for selecting thresholds can be formulated as follows:

$$\min_{\tau_{k,j} \geq 0} J(\tilde{p}), \quad J(\tilde{p}) := \sum_{k=0}^{N-1} \tilde{P}_k$$

s. to

C1. $\sum_{k=0}^{N-1} \log_2 M_k \text{BER}_k(\tau_k, \{P_{k,j}\}, M_k) = \text{BER}_0$

C2. $\sum_{k=0}^{N-1} \log_2 M_k = R_0$.

As in our D-CSIT based optimization, we will first solve (11) without accounting for C2. Upon substituting the expressions for $P_k$ and $\text{BER}_k$ into (11) and defining $\gamma_{k,j} := \beta_k P_{k,j+1}/\bar{g}_k$, the objective function and C1 for this relaxed problem can be written as:

$$J(\tilde{p}) = \sum_{k=0}^{N-1} \sum_{j=1}^{2^B-1} P_{k,j} \left( e^{-\gamma_{k,j} \tau_{k,j}} - e^{-\gamma_{k,j+1} \tau_{k,j+1}} \right),$$

(12)

$$\sum_{k=0}^{N-1} \frac{a \log_2 M_k}{R_0} \sum_{j=1}^{2^B-1} e^{-\gamma_{k,j} \tau_{k,j}} - e^{-\gamma_{k,j+1} \tau_{k,j+1}} = \text{BER}_0.$$

(13)

It can be easily shown that only locally optimal solutions can be ensured in our quantizer design due to lack of convexity. Nonetheless, the Lagrangian for minimizing (12) subject to (13) is given by

$$\mathcal{L} = \ldots + P_{k,j-1} \int_{\tau_{k,j-1}}^{\tau_{k,j}} f_{g_k}(g_k) dg_k + P_k \int_{\tau_{k,j}}^{\tau_{k,j+1}} f_{g_k}(g_k) dg_k + \ldots + \lambda_Q \left\{ \sum_k \frac{a \log_2 M_k}{R_0} \left[ \ldots + \int_{\tau_{k,j-1}}^{\tau_{k,j}} e^{-\beta_k g_k P_{k,j-1}} f_{g_k}(g_k) dg_k + \ldots \right] - \text{BER}_0 \right\}$$

(14)

where $\lambda_Q$ is found after substituting into C1. Upon defining $\psi(\gamma_{k,j}, \tau_{k,j}) := (1 + \gamma_{k,j} \tau_k \tau_{k,j}) e^{-\gamma_{k,j} \tau_k \tau_{k,j+1}}$, the necessary conditions $\partial \mathcal{L}/\partial \tau_{k,j} = 0$ and $\partial \mathcal{L}/\partial P_{k,j} = 0$, respectively, yield (for $k = 0, 1, \ldots N-1$ and $j = 1, \ldots 2^B-1$)

$$P_{k,j-1} - P_{k,j} - \lambda_Q \frac{a}{R_0} \log_2 M_k \right) \times \left[ e^{-\beta_k \gamma_{k,j} \tau_{k,j+1}} - e^{-\beta_k \gamma_{k,j} \tau_{k,j}} \right] = 0,$$

(15)
\[ e^{-\frac{\tau_{k,j}}{\bar{g}_k}} - e^{-\frac{\tau_{k,j+1}}{\bar{g}_k}} - \frac{\lambda_Q a \beta_k \log_2 M_k}{R_0 \bar{g}_k \gamma_k} \times \left| \psi(\gamma_{k,j}, \tau_{k,j}) - \psi(\gamma_{k,j}, \tau_{k,j+1}) \right| = 0. \]  

Eqs. (15) and (16) can be compactly written as nonlinear functions of the powers and thresholds as \( Z_1(P_{k,j-1}, P_{k,j}, \tau_{k,j}) = 0 \) and \( Z_2(P_{k,j}, \tau_{k,j}, \tau_{k,j+1}) = 0 \), respectively. We solve them to obtain \( \tau_{k,j} \) using the following off-line algorithm:

**Q1.** Sort the sub-channels in decreasing order as explained in the previous section but based on \( \bar{g}_k \) and generate a table of possible \( m \) candidates satisfying the rate constraint. Then, execute the following loop for each entry of \( m \):

- **For** \( k = 0 : N - 1 \)
  - **For** \( \tau_{k,1} = 0 : \tau_{\text{max}}, \) where \( \tau_{\text{max}} \) is found so that e.g., \( \Pr(\bar{g}_k > \tau_{\text{max}}) = 0.01; \)
  - Given a \( \tau_{k,1} \) value, solve (15) numerically (e.g., using the bisection method [9]) and obtain a candidate root \( P_{k,1} \) (recall \( P_{k,0} = 0 \)). With \( P_{k,1} \) given, solve (16) to determine \( \tau_{k,2} \). Similarly, keep alternating between (15) and (16) to eventually obtain \( \{P_{k,j}\} \) and \( \{\tau_{k,j}\} \) for all \( j = 1, \ldots, 2^B - 2 \). As these power and threshold values per sub-channel are found without using (16) with \( j = 2^B - 1 \), this equation can be used to test the feasibility of the candidate solution obtained.

  - End \( \tau_{k,1} \) loop.
  - End \( k \) loop.

At this point, we have a list of candidate solutions \( \tau_k, \bar{p} \). It remains to filter these solutions and accept only those achieving the average BER constraint in (13). End \( \lambda_Q \) loop.

**Q2.** For \( \lambda_Q = \lambda_Q_{\text{min}} : \lambda_Q_{\text{max}} \) where \( \lambda_Q_{\text{min}} \) and \( \lambda_Q_{\text{max}} \) values can be set from upper and lower bounds of \( \lambda_D \) in the D-CSIT scheme:

- **For** \( k = 0 : N - 1 \)
  - **For** \( \tau_{k,1} = 0 : \tau_{\text{max}}, \) where \( \tau_{\text{max}} \) is found so that e.g., \( \Pr(\bar{g}_k > \tau_{\text{max}}) = 0.01; \)
  - Given a \( \tau_{k,1} \) value, solve (15) numerically (e.g., using the bisection method [9]) and obtain a candidate root \( P_{k,1} \) (recall \( P_{k,0} = 0 \)). With \( P_{k,1} \) given, solve (16) to determine \( \tau_{k,2} \). Similarly, keep alternating between (15) and (16) to eventually obtain \( \{P_{k,j}\} \) and \( \{\tau_{k,j}\} \) for all \( j = 1, \ldots, 2^B - 2 \). As these power and threshold values per sub-channel are found without using (16) with \( j = 2^B - 1 \), this equation can be used to test the feasibility of the candidate solution obtained.

  - End \( \tau_{k,1} \) loop.
  - End \( k \) loop.

Among all feasible \( \tau \)'s, choose the one leading to the minimum \( J(\bar{p}) \) in (12).

In step Q1, each choice of \( \{M_k\}_{k=1}^{N_a} \) satisfies \( N_a \leq N \) which implies that the algorithm yields the thresholds only for the \( N_a \) sub-channels that are “best on the average”. Since per OFDM symbol, we may need thresholds for all sub-channels, numerical simulations have suggested a rule of thumb whereby for each \( j \), the ratio \( \tau_{k,j}/\bar{g}_k \) remains invariant for all \( k \).

**Remark 2:** In addition to thresholds, the off-line algorithm yields as a byproduct \( (m, \bar{p}) \) loadings that either one or both of them can be used in a simpler but suboptimal instantaneous or online loading that we will discuss towards the end of the next section.

**B. On-line Phase: Power and Bit Loading**

With the quantizer parameters (thresholds and thus regions) fixed, each gain realization \( g \) is estimated and quantized at the receiver to obtain the codeword \( j \). Feeding back this form of instantaneous Q-CSIT, the transmitter will find on-line the pair of loadings \( (m, \bar{p}) \) which minimize power subject to rate, and (what we could call conditioned on \( j \)) average BER per sub-channel. With \( \gamma_k := \beta_k P_k + 1/\bar{g}_k \), the latter can be expressed as:

\[
\text{BER}_{k,j} = \frac{\int_{\bar{g}_k}^{\tau_{k,j+1}} \text{BER}_k(P_k, \bar{g}_k)f_{\bar{g}_k}(\bar{g}_k)d\bar{g}_k}{\Pr(\bar{g}_k \in R_k,j)} = \frac{a(e^{-\gamma_k \tau_{k,j}} - e^{-\gamma_k \tau_{k,j+1}})}{\bar{g}_k \gamma_k(e^{-\gamma_k \tau_{k,j}/\bar{g}_k} - e^{-\gamma_k \tau_{k,j+1}/\bar{g}_k})},
\]

(17)

With this conditional average BER, our optimization problem can be formulated as follows:

\[
\min_{m \in M, \bar{p}(\bar{g})} J(\bar{p}), \quad J(\bar{p}) := \sum_{k=0}^{N-1} \text{BER}_k(P_k, \bar{g}_k) = \text{BER}_0
\]

(18)

C1. \( \sum_{k=0}^{N-1} \log_2 M_k \) \( \text{BER}_k(m, \tau_k, P_k, M_k) = \text{BER}_0 \)

C2. \( \sum_{k=0}^{N-1} \log_2 M_k = R_0 \).

Comparing (18) with (11), we infer that apart from the scalar in the denominator of (17), the two problems are basically identical. This scalar will alter the \( \lambda_Q \) multiplier, but otherwise, sorting the sub-channels according to their region indices and dropping the \( \tau_{k,1} \) loop, the off-line algorithm still can be applied to solve (18). Investigating the sufficient conditions for global optimality, here requiring \( \partial^2 J(\bar{p})/\partial P_k^2 \geq 0 \) and \( \partial^2 \text{BER}_{k,j}/\partial P_k^2 \geq 0 \), the first one can be readily verified while the latter yields \( \partial^2 \text{BER}_{k,j} = \frac{a \log_2 M_k}{R_0(e^{-\gamma_k \tau_{k,j}} - e^{-\gamma_k \tau_{k,j+1}})} \int_{\bar{g}_k}^{\tau_{k,j+1}} (\beta_k \bar{g}_k e^{-\gamma_k \bar{g}_k}) d\bar{g}_k \geq 0 \) where the last inequality follows since \( \tau_{k,j} < \tau_{k,j+1} \) and the integrand is non-negative.

In the spirit of Remark 2, the transmitter in the on-line phase can operate in any of the following modes (listed in decreasing power efficiency but also in decreasing complexity order):

**Tx A:** select optimally the set of active sub-carriers \( N_a \), bit loading \( m \), and the value of the Lagrange multiplier, \( \lambda \), to attain the global optimum \( \bar{p} \).

**Tx B:** select optimally \( N_a \) and \( \lambda \) while using a fixed \( m \) (no constellation adaptation).

**Tx C:** select optimally \( N_a \), but use fixed values for \( m \) and \( \lambda \).

**Tx D:** select optimally \( \lambda \), but use always the same \( N_a \) and \( m \).

Having considered D-CSIT and Q-CSIT, we look next into the possibility of exploiting only statistical knowledge of the channel. The resultant S-CSIT based OFDM is the least power efficient but also incurs the lowest complexity and feedback overhead.

**V. Optimization Based on S-CSIT**

Based on the channel pdf (which in our case depends only on \( \bar{g}_k \)), our objective in this section is to determine the power minimizing fixed pair of loadings \( (m, \bar{p}) \). The problem can be formulated as:

\[
\min_{m \in M, \bar{p}(\bar{g}) \geq 0} J(\bar{p}), \quad J(\bar{p}) := \sum_{k=0}^{N-1} \text{BER}_k(M_k, \bar{g}_k) = \text{BER}_0
\]

(19)

C1. \( \sum_{k=0}^{N-1} \log_2 M_k \) \( \text{BER}_k(P_k, M_k, \bar{g}_k) = \text{BER}_0 \)

C2. \( \sum_{k=0}^{N-1} \log_2 M_k = R_0 \),
where
\[
\overline{\text{BER}}_k(P_k, M_k, \bar{g}_k) = \int_0^\infty e^{-\beta_k P_k g_k} f_{g_k}(g_k) dg_k
\]
\[
= \frac{a}{1 + \beta_k P_k \bar{g}_k}.
\] (20)

Removing C2 as before, the KKT conditions on the relaxed problem yield for \(k \in \{0, \ldots, N - 1\},\)
\[
\bar{P}_k = \lambda_S \frac{\alpha \log_2 M_k}{R_0 \bar{g}_k} - \frac{1}{\bar{g}_k}.
\] (21)

where \(\lambda_S = \frac{1}{\overline{\text{BER}}_0} \sum_{k=0}^{K-1} \sqrt{\frac{\alpha \log_2 M_k}{R_0 \bar{g}_k}}.\) The sufficient conditions for optimality, \(\frac{\partial \bar{P}}{\partial P_k} \geq 0\) and \(\frac{\partial^2 \overline{\text{BER}}}{\partial P_k^2} \geq 0,\) are easily shown to hold true, thus ensuring global optimality. As discussed earlier, (21) must be calculated for all sorted candidates \(\mathbf{m}\) that meet the \(R_0\) constraint, selecting as final solution the one which requires the least power.

VI. NUMERICAL EXAMPLES

To numerically test our power-efficient designs, we considered an adaptive OFDM system with \(N = 64\) sub-carriers and \(B = 2\) feedback bits per sub-carrier (when Q-CSIT is utilized) with average gains \(\{\bar{g}_k\}_{k=0}^{N-1}\) allowed to vary over a 10dB range, and prescribed to achieve \(\text{BER}_0 = 10^{-3}\) at rate \(R_0 = 25\) bits (unless otherwise specified).

Test Case 1 (D-CSIT vs. Q-CSIT vs. S-CSIT in Power Efficiency and BER): For variable BER specifications, Figure 2 compares our preferred Q-CSIT scheme relative to the D-CSIT and S-CSIT benchmarks. We observe that Q-CSIT needs only around 1dB more power than D-CSIT and exhibits an average power gain of approximately 8, 15, and 24dB for \(\text{BER}_0 = 10^{-2}, 10^{-3},\) and \(10^{-4}\), respectively over the S-CSIT design. Compared to the no CSIT case, this gain almost doubles since the S-CSIT scheme has itself a power gain in the range of 15 to 25dB. Surprisingly, we also observe the almost-uniform reduction in power savings of the Q-CSIT based design compared to the D-CSIT based one across a wide range of BER values.

Test Case 2 (D-CSIT vs. Q-CSIT): Figure 3 quantifies the power loss when relying on Q-CSIT rather than D-CSIT for the different adaptive transmitters TxA-D. We observe that all four strategies TxA-D perform within 2 dB with respect to each other. In addition, the fully adaptive transmission (Tx-A) shows near-optimum power efficiency almost 1 dB away from the optimal D-CSIT benchmark.

Test Case 3 (Number of Quantization Regions): We finally investigated the effect of the number of feedback bits in the Q-CSIT scheme. Figure 4 demonstrates that power gains increase markedly when \(B\) increases from 1 to 2 bits, while a diminishing relative gain is expected for each additional increase in \(B\). It also shows that having \(B = 1\) entails only 2dB in power loss compared to the D-CSIT case which corresponds to \(B = \infty\).

VII. CONCLUSIONS

Under prescribed rate and error probability constraints, we optimized power-efficiency of wireless OFDM by relying on various forms of CSIT. Emphasis was placed on Q-CSIT which can become readily available because the receiver needs to feed back to the transmitter only a limited number of bits.
indexing the quantization region which the underlying channel falls into.

To choose a judicious form of Q-CSIT, we designed off-line optimal channel quantizers that rely on channel statistics (mean) to select quantization thresholds across OFDM sub-channels. We then designed an online optimum algorithm that allocates modulation and power levels across the sub-channels. The resultant optimized transceivers hold great potential since they were shown to enjoy about 15dB power savings relative to S-CSIT (and twice as much when compared to OFDM with no-CSIT available), while staying within 1dB from D-CSIT benchmark designs that are more suitable for slow fading wireless or wireline channels. It was also shown that having two feedback bits per sub-channel captures most of the gain loss due to quantization.

The promise power-efficient OFDM holds in the single-antenna point-to-point scenario considered here, motivates future research to explore its potential for power savings when multi-carrier modulation is used over multi-user and (possibly distributed) multi-antenna links in the context of ad hoc and wireless sensor networks.

REFERENCES