Asymptotically Optimal Cross-Layer Schemes for Relay Networks with Short-Term and Long-Term Constraints

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***Abstract*—Convex optimization and dual decomposition have been successfully used to design cross-layer resource allocation algorithms for cellular access networks. However, less effort has been devoted to design optimal algorithms for systems equipped with relay stations. Presence of relay stations renders the design of the access schemes more difficult and requires consideration of additional constraints. The present paper relies on a sum-utility constrained maximization framework to design cross-layer algorithms that guarantee diverse quality of service (QoS) and consider different forwarding strategies at the relay stations. One of the main challenges in the design is the *joint consideration* of both long-term (elastic) and short-term (real-time) constraints. Such constraints account for diverse delay QoS requirements and relay forwarding strategies. A two-step methodology is proposed to efficiently deal with this challenge. Specifically, for each time instant it applies: a) an approximate online method to *estimate* the multipliers for the long-term constraints and the corresponding primal variables (resources), and b) a classical iterative method to calculate the multipliers for the short-term constraints and the corresponding primal variables. Our approach incurs an arbitrarily small loss of optimality, and can accommodate both static and fading channels.**

***Index Terms*—Wireless relay networks, cross-layer design, network optimization, dynamic resource management, real-time communication.**

I. Introduction

Two key elements to satisfy the increasing demand of new multimedia communication services in wireless networks are: a) new architectures and topologies that render the access network more reliable and flexible and b) adaptive resource allocation algorithms that use channel and network state information to jointly optimize the system performance and guarantee quality of service (QoS). A simple but effective way to implement a) is the incorporation of relay stations (RSs), which can be effectively used to enlarge the cell coverage area, improve link reliability, increase capacity and alleviate fading effects; see, e.g., [1]–[4] and references therein for a recent state-of-the-art review. Regarding b), dynamic adaptation of resources of different layers (power, rates, routes) is a fundamental mechanism to optimize and guarantee QoS. Using dual decomposition techniques [5], [6] and adaptive control tools [7], many works have investigated the design of resource allocation algorithms in cellular systems for a broad range of setups (different types of fading models, services, access techniques, optimality criteria, QoS requirements, etcetera) and a few for multi-hop fading networks (see, e.g., [7], [8]). Fewer works have addressed the design of efficient (optimal) cross-layer resource allocation algorithms for relay systems; see [9]–[12] for notable exceptions. Two of the main challenges associated with the consideration of RSs are: i) the optimal design of schemes to access the channel is more difficult and ii) new constraints accounting for the forwarding strategy at the RSs need to be considered.

In this context, the present paper aims to use convex optimization to optimally design cross-layer resource allocation algorithms for *wireless relay-assisted* systems under *different operating conditions*. Users implement a relay selection strategy, so that they can choose between sending their packets directly to the BS or through an RS [1, Ch. 8]. Two schemes are considered for relaying the packets. The first one, known as instantaneous or synchronous relaying, assumes that the frame is divided into two slots. The first slot is used to send information from the users to the RSs, and the second slot to forward all the information received by the RSs during the first slot to the BS in a decode and forward fashion [1]. The second scheme allows for asynchronous (a.k.a. opportunistic) relaying. In this case, packets can be stored at the RS during several slots and transmitted to the users when the transmission conditions are more favorable [9]. Clearly, the second (asynchronous) scheme is more flexible at the cost of requiring more capabilities in the RS, so that it will achieve a higher long-term transmission rate; e.g., by exploiting the statistical variation of the channel or users' traffic. The remaining operating conditions are as follow. The BS, the RS and different users (flows) are considered to share orthogonally a set of parallel flat (fading) channels, which can be dynamically assigned to any of them. At each time slot, nodes can adapt their power and rate loadings per channel. Simple flow control mechanisms that allow for high arrival rates while keeping the network stable are implemented.

The optimal cross-layer resource allocation (flows, powers, rates, and channels) is obtained as the solution of a constrained optimization problem, which naturally takes into account flow-specific utility functions, individual QoS requirements, and the network operating conditions. The QoS guarantees consist of: i) maximum bit error rate (BER); ii) minimum long-term (elas-

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tic) and short-term (real-time) transmit rate (which is a meaningful feature of our algorithms [13], [14]); and iii) maximum long-term and short-term transmit-power [6]. It is shown that the resultant optimum dynamic resource allocation depends only on the current channel realization, and user-specific prices (Lagrange multipliers). A key to deal with the optimization problem is to find the allocation for each time instant in two steps. The first step accounts for the variables (resources) involved only in long-term/time-average/elastic constraints and the second step for the variables involved in short-term/time-instantaneous/real-time constraints. To keep the complexity of our algorithms low, the methods to find the corresponding multipliers are different. A classical iterative method is used to find the exact optimum multipliers associated with the instantaneous constraints, while an approximate method is developed to estimate online the multipliers associated with the average constraints. The latter method requires very low computational complexity, is robust to changes in the channel, and incurs an arbitrarily small loss in performance (optimality). Several works have employed related algorithms in wireless *fading* networks with operating conditions simpler than the ones in this paper [15]–[17]. In a nutshell, the contribution of the paper is twofold. First, we develop novel cross-layer resource allocation algorithms for relay access networks that were not considered before. Such algorithms are not mere extensions of the existing ones for cellular networks. Second, the (low-complexity) two-step strategy to deal with real-time and elastic constraints used in this paper, can also be used for systems with architectures and operating conditions very different to the ones considered here. Per instance, in cellular systems in which users transmit both real-time and delay-tolerant traffic.

The rest of the paper is structured as follows. The system setup, operating conditions and design approach are described in Secs. II and III. The constrained optimization problem for the case where only elastic traffic is present is formulated and solved in Sec. IV. The changes required to incorporate real-time traffic are discussed in Sec. V. Section VI briefly describes how the developed schemes can be extended to other meaningful scenarios. Numerical results and conclusions in Secs. VII and VIII wrap-up this manuscript. Due to space limitations, most mathematical manipulations and proofs are omitted. The key steps are identified and pertinent citations are provided.[1]

II. System setup and operating conditions

Consider for now the uplink of a cellular system where wireless terminals (users) can communicate either directly with a BS (single hop) or through an RS (two hops). The number of users and RSs are M and L, respectively. Variable $1 \leq m \leq M$ indexes users and variable $0 \leq l \leq L$ indexes RSs, with $l = 0$ referring to the BS. When a user m decides to use RS l, binary variable b is used to index each of the possible two hops. Specifically, $b = 0$ is used for the user-RS hop and $b = 1$ for the RS-BS hop. A pair (l, b) will be referred to as *hop* and the set containing all possible hops is denoted as $\mathcal{S} := \{(l, b),\ l \geq 1,\ b = 0, 1\} \bigcup \{(0, 0)\}$. The overall bandwidth W is divided into K orthogonal channels, each with bandwidth W/K small enough to ensure that the channel is flat, i.e., non-selective. The wireless channel gain for a transmission from user m on hop (l, b) using channel k is characterized by the square magnitude $h^{l,b}_{m,k}$, which is assumed normalized by the power noise. Note that $h^{l,1}_{m,k} := h^{l,1}_{k} \quad \forall m$. The vector containing the channel gains for all users, channels and hops has length $(2L + 1)MK$ and it is denoted by $\mathbf{h} := \{h^{l,b}_{m,k},\ m = 1, \ldots, M,\ k = 1, \ldots, K,\ (l, b) \in \mathcal{S}\}$. The user-hop triplet (m, l, b) is referred to as *link* and the set containing all possible links is denoted as $\mathcal{T}$.

We consider an adaptive system in which the allocation of resources (transmit power and rates –physical layer–, scheduling –multiple access layer–, and arrival rates –transport layer–) can be modified at every time slot (n). In the following, we introduce the variables pertaining to each layer and the constraints (relationships) that such variables need to satisfy.

Multiple access layer: In this work, users implement a relay selection strategy [1], so that *links* at the outset are scheduled to access simultaneously but *orthogonally* in time (or frequency) any of the channels. To describe access quantitatively, let $w^{l,b}_{m,k}[n] \in [0, 1]$ denote the nonnegative fraction of time (or channel bandwidth) that the link (m, l, b) is scheduled to transmit over channel k during the slot n. Since transmissions interfere each other, no frequency reuse is allowed and it must hold that

$$\sum_{m=1}^{M} \sum_{(l,b)\in\mathcal{S}} w^{l,b}_{m,k}[n] \leq 1, \quad \forall k,\ \forall n. \tag{1}$$

This way, if $w^{0,0}_{m,k}[n] = 0.9$ and $w^{l,0}_{m',k}[n] = 0.1$, the direct link between the user m and the BS on the kth channel is active during 90% of the duration of slot n, the link between the user m' and the RS l during the remaining 10%, and no other link is scheduled. Different from our formulation, some works impose that $w^{l,b}_{m,k}[n] \in \{0, 1\}$, so that $w^{l,b}_{m,k}[n]$ acts as an indicator variable. As it will be apparent in the next section, the specific scheduling constraints that variables $w^{l,b}_{m,k}[n]$ need to satisfy may change depending on the operating conditions at the RS and the properties (requirements) of the traffic. As mentioned before, this paper focuses on two-hop relaying (meaning that an RS cannot forward the packets to another RS) and does not allow for frequency reuse. Relaxation of these conditions (and analysis of the modifications required in the multiple access layer to account for them) is an issue of interest, but it is out of the scope of this manuscript.

Physical layer: The resources adapted are power and rate per link and channel. Let $p^{l,b}_{m,k}[n]$ and $r^{l,b}_{m,k}[n]$ denote, respectively, the instantaneous power and rate transmitted over the link (m, l, b) and channel k during the slot n provided that $w^{l,b}_{m,k}[n] = 1$. (If $w^{l,b}_{m,k}[n] < 1$ the effective power transmitted during the slot n is $p^{l,b}_{m,k}[n]w^{l,b}_{m,k}[n]$.)

With $\tilde{p}^{U}_{m}$ denoting the maximum *average* (long-term) power

[1] *N*otation: x^* denotes the optimal value of x; $\vee$ and $\wedge$ the logical "or" and "and" operators; $:=$ definition; $\log(x)$ the logarithm to the base e of x; and $[x]^b_a$ the projection of x onto the $[a, b]$ interval, i.e., $[x]^b_a = \min\{\max\{a, x\}, b\}$. For a function $f(\cdot)$, $(f)^{-1}(\cdot)$ denotes its inverse and $\dot{f}(\cdot)$ its derivative. To help readability the most significant notation introduced in the ensuing sections is summarized in Table III.

user m can transmit, the following constraint needs to hold

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{l=0}^{L}p_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n]\leq\check{p}_m^U,\ \forall m. \quad (2)$$

Similarly, with $\check{p}_l^R$ denoting the maximum average power the RS l can transmit, it holds that

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{m=1}^{M}p_{m,k}^{l,1}[n]w_{m,k}^{l,1}[n]\leq\check{p}_l^R. \quad (3)$$

Under maximum BER constraints, power $p_{m,k}^{l,b}[n]$ is coupled with the corresponding rate $r_{m,k}^{l,b}[n]$. To be more specific, let $\check{e}_m$ denote the maximum BER value and suppose that users are implementing coded QAM modulations. Then, the BER for a given power-rate pair can be accurately approximated as $e_m = 0.2\exp([-p_{m,k}^{l,b}[n]h_{m,k}^{l,b}[n]g_{m,k}]/[2^{r_{m,k}^{l,b}[n]}-1])$, where $g_{m,k}$ stands for the coding gain of channel coding scheme implemented [18]. Setting the left hand side of the previous equality to $\check{e}_m$, it readily follows that $r_{m,k}^{l,b}[n]$ can be written as a function of $p_{m,k}^{l,b}[n]$. The rate-power function will be denoted as $r_{m,k}^{l,b}[n] = C_{m,k}^{l,b}(\mathbf{h}, p_{m,k}^{l,b}[n])$. Note that for the previous example $C_{m,k}^{l,b}(\mathbf{h}, p_{m,k}^{l,b}[n]) = \log_2(1 + h_{m,k}^{l,b}p_{m,k}^{l,b}[n]/\Gamma_{m,k}^{l,b})$, where $\Gamma_{m,k}^{l,b} := \log(\check{e}_m/0.2)/g_{m,k}$ represents the so-called SNR-gap relative to the well-known Shannon's capacity formula [18]. Nonetheless, the results in the paper hold for any rate-power function $C_{m,k}^{l,b}(\mathbf{h},\cdot)$ which is increasing, strictly concave and differentiable.

Transport and network layers: Packets are generated exogenously at higher layers. Packet streams will be referred to as flows, each user having one flow (the formulation can be easily extended later to accommodate more than one flow per user). The average arrival rate of exogenous information of flow m is denoted by $\bar{a}_m$, which is a parameter to be designed. *Users* transmitting *elastic* flows are equipped with queues (buffers) where incoming packets are stored before transmission. Each of such queues must satisfy the following necessary condition:

$$\bar{a}_m/(1-\beta)\leq\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{l=0}^{L}r_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n] \quad (4)$$
$$\forall m,\ 0\leq\beta<1.$$

The latter is typically known as *average* flow-conservation condition. Parameter β is a penalizing coefficient accounting for the rate-loss due to the overhead of packet headers arriving at the transport layer. Equations (1)-(4) will act as constraints in the optimization problem presented in Sec. IV.

III. Design approach

The optimal resource allocation will be obtained as the solution of a constrained optimization problem. To properly formulate that problem we need first to identify: a) the optimization (design) variables, b) the objective to be optimized, and c) the constraints to be satisfied.

a) The resources at the different layers are the *optimization variables*, namely $w_{m,k}^{l,b}[n]$, $p_{m,k}^{l,b}[n]$ and $\bar{a}_m$. Note that there is no need to optimize over $r_{m,k}^{l,b}[n]$, because under a maximum BER constraint $r_{m,k}^{l,b}[n]$ can be replaced with $C_{m,k}^{l,b}(\mathbf{h}, p_{m,k}^{l,b}[n])$; thus, the optimum value of $p_{m,k}^{l,b}[n]$ readily yields the optimum value of $r_{m,k}^{l,b}[n]$.
b) The *objective* will entail concave and increasing so-called *utility functions* $U_m(\cdot)$, that are commonly used in resource allocation tasks (not only restricted to communication systems), and account for the "social" utility (reward) that a specific resource (here $\bar{a}_m$) gives rise to; see e.g., [7], [8], and references therein. Utility functions $U_m(\bar{a}_m)$ are chosen to be increasing (so that solutions which allow for higher arrival rates are promoted) and concave (so that the marginal utility for each user terminal diminishes as its rate increases), which offers a mean to effect fairness among different users [7], [8]. Utilities are user dependent and should be selected based on the type of traffic to be flown.
c) The relationships and conditions presented in the previous section (1)-(4) are explicitly imposed as *constraints*. Moreover, to effect additional QoS, a *minimum average* arrival rate $\check{a}_m$ is guaranteed for certain users so that

$$\bar{a}_m\geq\check{a}_m,\ m=1,...,M. \quad (5)$$

Users with no rate guarantees will set $\check{a}_m$ to zero. Additional constraints will be introduced in the next sections to account for specific operating conditions of the setups considered.

For notational convenience, throughout the following sections three set of users (flows) will be considered: $\mathcal{M}_I$, $\mathcal{M}_{II}$ and $\mathcal{M}_{III}$. Users will be assigned to these sets based on two criteria: a) whether the traffic has real-time requirements and b) whether the RS has to retransmit the received packets instantaneously (i.e., during the slot after the one in which were received). This way, set $\mathcal{M}_I$ contains elastic flows (negative answer for a) which allow for an asynchronous relaying strategy (negative answer for b). The latter implies that packets can be stored at the RS during several slots and retransmitted when the transmission conditions are more favorable [9]. Set $\mathcal{M}_{II}$ contains elastic flows (negative answer for a) that do not allow for an asynchronous relaying strategy (positive answer for b). The latter can be either because the traffic is connection oriented or due to hardware limitations on the RS [1]. Set $\mathcal{M}_{III}$ is formed by the flows with real-time requirements and which require instantaneous retransmissions (positive answer for both a and b). Clearly, real-time flows can never allow for an asynchronous relaying strategy, so there is no need for defining a fourth set.

IV. Non-real time traffic

This section formulates and solves the optimal resource allocation problem for systems with elastic flows ($\mathcal{M}_I$ and $\mathcal{M}_{II}$). To simplify exposition, real-time flows ($\mathcal{M}_{III}$) are incorporated in Sec. V.

For flows in $\mathcal{M}_I$, packets received at an RS are not relayed instantaneously. Hence, RSs need to have per-flow queues satisfying the following necessary *average* condition for stability [cf. (4)]

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K} r_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n] \leq \\ \leq \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K} r_{m,k}^{l,1}[n]w_{m,k}^{l,1}[n],\ m\in\mathcal{M}_I,\ l\geq 1. \tag{6}$$

For flows in $\mathcal{M}_{II}$, the frame is divided into two slots: the first slot is devoted to transmissions from users to the RS and in the second slot the RS relays the information to the BS. To facilitate exposition, without loss of optimality we will assume that: i) transmissions from users in $\mathcal{M}_{II}$ to the RS take place when the index n is an odd number; ii) packets from users in $\mathcal{M}_{II}$ are retransmitted from the RS to the BS when n is an even number; and iii) the allocation of resources is performed every frame (two slots), meaning that the allocation for slots n and $n+1$ is found at the same time and only if n is an odd number. No further constraints are considered, so that one-hop (direct) transmissions between the users and the BS can take place at any n. With this operational assumptions, the flows in $\mathcal{M}_{II}$ do not have to satisfy average flow-conservation constraint (6), but its *instantaneous* counterpart

$$\sum_{k=1}^{K} r_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n] \leq \sum_{k=1}^{K} r_{m,k}^{l,1}[n+1]w_{m,k}^{l,1}[n+1] \\ m\in\mathcal{M}_{II},\ l\geq 1,\ \forall \text{ odd } n. \tag{7}$$

Note that (7) is more restrictive than (6). This entails that: i) flows in $\mathcal{M}_{II}$ will satisfy (6) too, and ii) the performance (utility and rate) will be higher for the flows in $\mathcal{M}_I$. Clearly, (7) requires that all packets received by the RSs have to be forwarded within the same frame. Formulations allowing for retransmissions within a given window are also possible. See Sec. V for further discussion on this issue.

Had only flows in $\mathcal{M}_I$ been present, the scheduling variables would just need to satisfy (1). However, when both types of flows are present, they also need to satisfy:

$$w_{m,k}^{l,0}[n]=0\ \ \forall k,\ m\in\mathcal{M}_{II},\ \forall \text{ even } n \quad \text{and} \\ w_{m,k}^{l,1}[n]=0\ \ \forall k,\ m\in\mathcal{M}_{II},\ \forall \text{ odd } n. \tag{8}$$

Constraints (6), (7) and (8) are not present in existing cellular access networks, and are the main challenge for the design of the resource allocation algorithms in this paper.

Taking into account these conditions, the optimal allocation is obtained by solving the following sum-utility constrained maximization:

$$\mathsf{P}^* := \max_{\substack{\{\forall \bar{a}_m, \forall w_{m,k}^{l,b}[n],\\ \forall p_{m,k}^{l,b}[n]\}}} \sum_{m=1}^{M} U_m\left(\bar{a}_m\right) \tag{9a}$$

$$\text{subject to}: \ (2),\ (3),\ (4),\ (5),\ (6) \tag{9b}$$

$$(7),\ (8). \tag{9c}$$

The solution of (9) will be pursued in the next section, but several remarks are due before that. First, conditions in (9b) are elastic constraints (meaning that they do not have to hold for every slot, but only on average), while conditions in (9c) are real-time constraints (meaning that they need to hold for each and every slot n). This difference is relevant because the approach to deal with average constraints and the one to deal with instantaneous constraints will be different. Second, cross-layer attribute of the resource allocation in (9) is apparent because variables of different layers are jointly optimized. Third, if the requirements in (9b) are too strong, the optimization in (9) could be infeasible. Although infeasibility is readily detectable, the design of admission control mechanisms to render the problem feasible is a difficult problem (often NP-hard), and goes beyond the scope of this work.

A. Optimal cross-layer allocation

The problem in (9) can be readily transformed into a convex one by following an approach similar to that in [16]. A key step to solve the optimization problem is to handle the constraints (9b) and (9c) separately. All average constraints will be guaranteed using a dual approach, so that a Lagrange multiplier (dual variable) is associated with each of them. Because these constraints do not vary with time, the same holds for the *optimum* value of the corresponding multipliers. Assuming that the value of such multipliers is known, for each slot n we first find the optimum value of variables involved only in average constraints, and then we solve a simpler problem for which only the instantaneous constraints (9c) remain. To solve the instantaneous problem, the value of the average multipliers and variables is given, and some of the constraints are dualized (so that the corresponding instantaneous multipliers need to be found), while others are guaranteed using alternative methods. In the remaining of this subsection, we present the optimal solution for average and instantaneous variables as a function of the Lagrange multipliers.

Solution for variables involved only in average constraints: Let $\pi_m^{0,0}$, $\pi^{l,1}$, $\rho_m^{0,0}$, α_m and $\rho_m^{l,1}$ denote the Lagrange multipliers associated with the *average* constraints in (2), (3), (4), (5), and (6), respectively. For notational convenience, define also $\rho_m^{l,0} := \left[\rho_m^{0,0}-\rho_m^{l,1}\right]_0^{\infty}$ for $m\in\mathcal{M}_I$ and $l\geq 1$, and $\pi_m^{l,1} := \pi^{l,1}$ and $\pi_m^{l,0} := \pi_m^{0,0}$ for all m and $l\geq 1$. Collect all the multipliers in a vector $\boldsymbol{\lambda}$. All the primal variables are collected in vector $\mathbf{y} := [\mathbf{y}_1, \mathbf{y}_2]$, with $\mathbf{y}_1$ collecting the variables involved only in average constraints and $\mathbf{y}_2$ the remaining variables. The set of links $\mathcal{T}$ is split into two disjoint sets $\mathcal{T}_1$ and $\mathcal{T}_2$, so that $\mathcal{T}_1 := \{\ (m,l,b):\ \ \big(m\in\mathcal{M}_I\big)\ \vee\ \big((m\in\mathcal{M}_{II})\ \wedge\ (l,b)=(0,0)\big)\}$ and $\mathcal{T}_2$ contains all other elements in $\mathcal{T}$. Note that $\mathbf{y}_1$ collects all average arrival rates and the powers and rates for links in $\mathcal{T}_1$, while $\mathbf{y}_2$ collects all scheduling variables and the powers and rates for links in $\mathcal{T}_2$. With these notational conventions, the Lagrangian of the problem of maximizing (9a) subject to (9b) is

To find the optimal $\mathbf{y}$ which minimizes $\mathcal{L}(\mathbf{y},\boldsymbol{\lambda})$ we will rely on the fact that the optimum value of $\mathbf{y}_1$ does not depend on $\mathbf{y}_2$. As a result, the minimization can be split into two steps. First, $\mathcal{L}(\mathbf{y},\boldsymbol{\lambda})$ is minimized w.r.t. $\mathbf{y}_1$, yielding $\mathbf{y}_1^*(\boldsymbol{\lambda})$. Second, $\mathbf{y}_1^*(\boldsymbol{\lambda})$ is substituted into $\mathcal{L}(\mathbf{y},\boldsymbol{\lambda})$, yielding $\mathcal{L}(\mathbf{y}_2,\boldsymbol{\lambda}) := \mathcal{L}([\mathbf{y}_1^*(\boldsymbol{\lambda}),\mathbf{y}_2],\boldsymbol{\lambda})$,which is then minimized w.r.t. $\mathbf{y}_2$. To address the first step, let $(\dot{U}_m)^{-1}(\cdot)$ and $(\dot{C}_{m,k}^{l,b})^{-1}(\mathbf{h},\cdot)$

$$\mathcal{L}(\mathbf{y},\boldsymbol{\lambda}) := \sum_{m=1}^{M}\Big(U_m(\bar{a}_m)+\alpha_m(\check{a}_m-\bar{a}_m)\Big) - \sum_{l=1}^{L}\pi^{l,1}\left(\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{m=1}^{M}p_{m,k}^{l,1}[n]w_{m,k}^{l,1}[n]-\check{p}_l^R\right) \tag{10a}$$

$$-\sum_{m=1}^{M}\pi_m^{0,0}\left(\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{l=0}^{L}p_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n]-\check{p}_m^U\right) \tag{10b}$$

$$-\sum_{m=1}^{M}\rho_m^{0,0}\left(\bar{a}_m/(1-\beta)-\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}\sum_{l=0}^{L}r_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n]\right) \tag{10c}$$

$$-\sum_{m\in\mathcal{M}_I}\sum_{l=1}^{L}\rho_m^{l,1}\left(\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}r_{m,k}^{l,0}[n]w_{m,k}^{l,0}[n]-\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=1}^{K}r_{m,k}^{l,1}[n]w_{m,k}^{l,1}[n]\right). \tag{10d}$$

denote the inverse function of the derivative of $U_m(\cdot)$ and $C_{m,k}^{l,b}(\mathbf{h},\cdot)$, respectively; and remember that x^* stands for the optimum value of a given variable x. Using this notational conventions, it can be shown that the optimal $\mathbf{y}_1$ minimizing the Lagrangian in (10) for a given $\boldsymbol{\lambda}$ is[2]

$$\bar{a}_m^*(\boldsymbol{\lambda}) = \left[(\dot{U}_m)^{-1}(\rho_m^{0,0}/(1-\beta)-\alpha_m)\right]_0^{\infty},\ \forall m \tag{11}$$

$$p_{m,k}^{l,b*}(n,\boldsymbol{\lambda}) = \left[(\dot{C}_{m,k}^{l,b})^{-1}\left(\mathbf{h},\pi_m^{l,b}/\rho_m^{l,b}\right)\right]_0^{\check{p}^k},\ (m,l,b)\in\mathcal{T}_1 \tag{12}$$

$$r_{m,k}^{l,b*}(n,\boldsymbol{\lambda}) = C_{m,k}^{l,b}(\mathbf{h},p_{m,k}^{l,b*}(n,\boldsymbol{\lambda})),\ (m,l,b)\in\mathcal{T}_1. \tag{13}$$

Note that the value the optimum power was projected onto the interval $[0,\check{p}^k]$, so that the transmitter power can never be greater than $\check{p}^k$. Such upperbound represents the maximum instantaneous power the terminals can transmit on channel k (spectrum-mask constraints). This condition was not explicitly imposed as a constraint because it can be straightforwardly guaranteed via the projection in (12).

Solution for variables involved in instantaneous constraints: The method to find the optimum variables $\mathbf{y}_2$ (rate and power loadings for links in $\mathcal{T}_2$, and scheduling for all links) is a bit more intricate. The basic idea is to write $\mathcal{L}(\mathbf{y}_2,\boldsymbol{\lambda})$, which is an aggregate function, as $\mathcal{L}(\mathbf{y}_2,\boldsymbol{\lambda}) = c+\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\mathcal{L}_n(\mathbf{y}_2[n],\boldsymbol{\lambda})$, where c accounts for the constant terms that do not depend on $\mathbf{y}_2$ and $\mathcal{L}_n(\mathbf{y}_2[n],\boldsymbol{\lambda})$ stands for the terms in $\mathcal{L}(\mathbf{y}_2,\boldsymbol{\lambda})$ corresponding to slot n. Based on this notation, at every slot n we need to solve an optimization problem that: a) minimizes $\mathcal{L}_n(\mathbf{y}_2[n],\boldsymbol{\lambda})$ and b) satisfies the instantaneous constraints in (1), (7) and (8). To formulate such a problem, let us first define the functional

$$\varphi(\rho_m^{l,b},\pi_m^{l,b},r_{m,k}^{l,b}[n],p_{m,k}^{l,b}[n]) := \rho_m^{l,b}r_{m,k}^{l,b}[n]-\pi_m^{l,b}p_{m,k}^{l,b}[n]. \tag{14}$$

Intuitively, $\varphi(\rho_m^{l,b},\pi_m^{l,b},r_{m,k}^{l,b}[n],p_{m,k}^{l,b}[n])$ can be viewed as a link-quality indicator (the higher the indicator, the higher the reward of activating the specific link). Mathematically, $\varphi(\rho_m^{l,b},\pi_m^{l,b},r_{m,k}^{l,b}[n],p_{m,k}^{l,b}[n])$ is the contribution to $\mathcal{L}(\mathbf{y},\boldsymbol{\lambda})$ in (10) if the corresponding link is activated during slot n; i.e., if $w_{m,k}^{l,b}[n]=1$. The optimum variables $\mathbf{y}_2$ at instant n are then obtained as the solution of the following convex optimization problem:

The approach to solve this problem is to deal with the constraints in (15) in a different manner. More specifically, the flow conservation constraints in (7) will be dualized, while the constraints in (1) and (8) will not. Let $\rho_m^{l,1}[n]$ be the (instantaneous) Lagrange multiplier associated with (7) and let $\rho_m^{l,1*}[n]$ denote the *optimal* value of such multiplier; i.e., the value for which the corresponding constraint is satisfied and the primal solution is optimal. Note that the optimum value is different for each n because the constraint (7) needs to hold at every slot n. Assuming that at time n the optimum value of the multipliers $\rho_m^{l,1*}[n]$ for $m\in\mathcal{M}_{II}$ is known, the two facts described next hold. Fact 1: the optimal power (hence rate) allocation for links in $\mathcal{T}_2$ and instant n is

$$p_{m,k}^{l,b*}(n,\boldsymbol{\lambda}) = \left[(\dot{C}_{m,k}^{l,b})^{-1}\left(\mathbf{h},\pi_m^{l,b}/\rho_m^{l,b*}[n]\right)\right]_0^{\check{p}^k},\ (m,l,b)\in\mathcal{T}_2 \tag{16}$$

$$r_{m,k}^{l,b*}(n,\boldsymbol{\lambda}) = C_{m,k}^{l,b}(\mathbf{h},p_{m,k}^{l,b*}(n,\boldsymbol{\lambda})),\ (m,l,b)\in\mathcal{T}_2, \tag{17}$$

where with a slight abuse of notation, $\rho_m^{l,0*}[n] := \rho_m^{l,0}-\rho_m^{l,1*}[n]$ for $m\in\mathcal{M}_{II}$. The only difference between (16)-(17) [which hold for $(m,l,b)\in\mathcal{T}_2$] and (12)-(13) [which hold for $(m,l,b)\in\mathcal{T}_1$] is that $\rho_m^{l,b}$ has been replaced with $\rho_m^{l,b*}[n]$. Fact 2: The optimal scheduling variables $\{w_{m,k}^{l,b*}(n,\boldsymbol{\lambda})\}$ for instant n can be found as the solution of the following problem

The main differences between (18) and (15) are: i) only the scheduling coefficients are optimization variables in (18) and ii) the instantaneous flow conservation constraint is not present in (18). The first difference implies that (18) is a *linear* program (LP), the second that the problem can be separated across channels (meaning that the optimal scheduling for each channel can be found separately).

B. Estimating the Lagrange multipliers

Methods to calculate the multipliers (both for the multipliers associated with the instantaneous constraints and for the ones associated with the average constraints) are presented in this section.

Multipliers associated with instantaneous constraints: To find the resource allocation that exactly satisfies the average constraints and maximizes (15), the optimal value for $\rho_m^{l,1}[n]$

[2]Mathematical derivations are omitted due to space limitations. The expressions in (11)-(13) were obtained by setting the partial derivatives of $\mathcal{L}(\mathbf{y},\boldsymbol{\lambda})$ w.r.t. $\mathbf{y}_1$ to zero and solving for the corresponding variables. A full proof for the *cellular* version of the problem in (9) –which does not consider constraints (7), (6) and (8)– can be found in [6].

$$\max_{\substack{\{w_{m,k}^{l,b}[n],\ (m,l,b)\in\mathcal{T}\}_{k=1}^{K},\\ \{p_{m,k}^{l,b}[n],\ (m,l,b)\in\mathcal{T}_2\}_{k=1}^{K}}} \sum_{k=1}^{K}\Bigg(\sum_{(m,l,b)\in\mathcal{T}_1} w_{m,k}^{l,b}[n]\varphi(\rho_m^{l,b},\pi_m^{l,b},r_{m,k}^{l,b*}(n,\boldsymbol{\lambda}),p_{m,k}^{l,b*}(n,\boldsymbol{\lambda}))$$
$$+\sum_{(m,l,b)\in\mathcal{T}_2} w_{m,k}^{l,b}[n]\varphi(0,\pi_m^{l,b},r_{m,k}^{l,b}[n],p_{m,k}^{l,b}[n])\Bigg) \quad \text{subject to}: \quad (1),\ (7),\ (8). \tag{15a}$$

$$\max_{\{w_{m,k}^{l,b}[n],\ (m,l,b)\in\mathcal{T}\}_{k=1}^{K}} \sum_{k=1}^{K}\Bigg(\sum_{(m,l,b)\in\mathcal{T}_1} w_{m,k}^{l,b}[n]\varphi(\rho_m^{l,b},\pi_m^{l,b},r_{m,k}^{l,b*}(n,\boldsymbol{\lambda}),p_{m,k}^{l,b*}(n,\boldsymbol{\lambda}))$$
$$+\sum_{(m,l,b)\in\mathcal{T}_2} w_{m,k}^{l,b}[n]\varphi(\rho_m^{l,b*}[n],\pi_m^{l,b},r_{m,k}^{l,b*}(n,\boldsymbol{\lambda}),p_{m,k}^{l,b*}(n,\boldsymbol{\lambda}))\Bigg) \quad \text{subject to}: \quad (1),\ (8). \tag{18a}$$

needs to be found and substituted back into (16)-(18). Our approach to find $\rho_m^{l,1*}[n]$ is to use an iterative algorithm. Specifically, a classical dual subgradient method with diminishing stepsize is proposed [19, Ch. 4 and 6]. Before developing the iterations for the algorithm, we need to introduce some notation. Let i denote an iteration index and $\rho_m^{l,1}[n,i]$ the value of the multiplier $\rho_m^{l,1}[n]$ corresponding to iteration i. Moreover let $r_{m,k}^{l,b*}[n,i]$ and $w_{m,k}^{l,b*}[n,i]$ denote the value for the rates and scheduling at iteration i, such values are the solution in (16)-(18) after substituting for $\rho_m^{l,1*}[n]=\rho_m^{l,1}[n,i]$. For mathematical convenience define also the effective rate as $\tilde{r}_m^{l,b*}[n,i] := \frac{1}{2}\sum_{j=0}^{1}\sum_{k=1}^{K} r_{m,k}^{l,b*}[n+j,i]w_{m,k}^{l,b*}[n+j,i]$. Let $\mu[i]$ denote a diminishing stepsize and consider the iterations

$$\rho_m^{l,1}[n,i+1]=\left[\rho_m^{l,1}[n,i]+\mu\left(\tilde{r}_m^{l,0*}[n,i]-\tilde{r}_m^{l,1*}[n,i]\right)\right]_0^{\infty} \tag{19}$$

If the stepsize satisfies $\sum_{i=1}^{\infty}\mu[i]=\infty$ and $\sum_{i=1}^{\infty}(\mu[i])^2<\infty$, then it can be rigorously established that the value of $\rho_m^{l,1}[n,i+1]$ converges to $\rho_m^{l,1*}[n]$ as i grows large. In practical systems, the *exact* optimum value of $\rho_m^{l,1*}[n]$ may not be found (mostly, due to computational limitations). However, simulations will show that convergence to a small neighborhood of $\rho_m^{l,1*}[n]$ occurs for a small value of i (for large values of n, 2-5 iterations may be enough). The Lipschitz continuity of the rate-power function guarantees that small errors in the multipliers will give rise to small errors in power and rates. However, that is not the case for $w_{m,k}^{l,b*}[n,i]$, which may be discontinuous at $\rho_m^{l,1}[n,i]=\rho_m^{l,1*}[n]$ (see the discussion on the lack of Lipschitz continuity of scheduling variables in [16]). As a result, the instantaneous flow constraints (**??**) may be violated. Different alternatives to prevent this violation can be implemented: i) use the asymptotically optimum closed-form Lipschitz continuous scheduling proposed in [16]; ii) use a primal averaging approach to find the (approximate) optimum value of the scheduling variables [20], [21]; iii) use the approximate value of the multipliers to calculate powers and rates but incorporate the constraints (7) into the problem in (18) (optimization is carried only over the scheduling constraints). The latter approach will slightly increase the complexity required to find the optimum scheduling coefficients. That is the case because when constraints (7) are incorporated into (18), the resultant LP problem cannot be separated across channels. Our simulations will be run using the method in i).

Multipliers associated with average constraints: An iterative method to find the optimum value of the multipliers was presented in the previous section. Differently, an online approach that for each slot n estimates the value of $\boldsymbol{\lambda}$ is used here for the multipliers associated with the average constraints. Under this approach, the online estimate, call it $\hat{\boldsymbol{\lambda}}[n]$, may never converge to the exact $\boldsymbol{\lambda}^*$, but remains sufficiently close to it; see e.g., [15] for a similar strategy for systems operating over fast time-varying fading channels. The main motivation behind this online estimate is that the computational complexity required is much lower than that of the classical iterative methods. Classical methods require running hundreds/thousand of iterations off-line during the initialization phase. For each iteration, either one or two nested loops have to be implemented (one for the short-term optimization and another one if fading is present and a Monte Carlo approach is required). This computational burden may be too high in some practical systems. On top of being computationally affordable, the online schemes will be able to cope with changes in the system setup (number of users, QoS constraints and channel gains). Next, we present the online schemes and then their performance is characterized (specifically, we will show that the resource allocation is asymptotically feasible and incurs an arbitrarily small loss of performance).

To help readability, let define the effective power as $\tilde{p}_m^{l,b*}(n,\hat{\boldsymbol{\lambda}}[n]) = \frac{1}{2}\sum_{j=0}^{1}\sum_{k=1}^{K} p_{m,k}^{l,b*}(n+j,\hat{\boldsymbol{\lambda}}[n])w_{m,k}^{l,b*}(n+j,\hat{\boldsymbol{\lambda}}[n])$ and define $\tilde{r}_m^{l,b*}(n,\hat{\boldsymbol{\lambda}}[n])$ analogously. With μ denoting a *small* and *constant* stepsize, the average Lagrange multipliers at slot $n+2$ are estimated as:

where the value of $\hat{\boldsymbol{\lambda}}[0]$ is generated randomly and (20d) is run only for users in $\mathcal{M}_I$. The primal variables in (20a)-(20e), namely, powers, rates, scheduling fractions and flow rates, are found by substituting $\hat{\pi}_m^{0,0}[n]$, $\hat{\pi}^{l,1}[n]$, $\hat{\rho}_m^{0,0}[n]$, $\hat{\rho}_m^{l,1}[n]$, and $\hat{\alpha}_m[n]$ into (11)-(18). Those will be referred to as online primal variables.

Convergence, feasibility and optimality of the online

$$\hat{\pi}_m^{0,0}[n+2] := \left[\hat{\pi}_m^{0,0}[n] - \mu\left(\check{p}_m^U - \sum\nolimits_{l=0}^{L}\tilde{p}_m^{l,0*}(n,\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{20a}$$

$$\hat{\pi}^{l,1}[n+2] := \left[\hat{\pi}^{l,1}[n] - \mu\left(\check{p}_l^R - \sum\nolimits_{m=1}^{M}\tilde{p}_m^{l,1*}(n,\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{20b}$$

$$\hat{\rho}_m^{0,0}[n+2] := \left[\hat{\rho}_m^{0,0}[n] + \mu\left(\bar{a}_m^*(\hat{\boldsymbol{\lambda}}[n])/(1-\beta) - \sum\nolimits_{l=0}^{L}\tilde{r}_m^{l,0*}(n,\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{20c}$$

$$\hat{\rho}_m^{l,1}[n+2] := \left[\hat{\rho}_m^{l,1}[n] + \mu\left(\tilde{r}_m^{l,0*}(n,\hat{\boldsymbol{\lambda}}[n]) - \tilde{r}_m^{l,1*}(n,\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{20d}$$

$$\hat{\alpha}_m[n+2] := \left[\hat{\alpha}_m[n] + \mu\left(\check{a}_m - \bar{a}_m^*(\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{20e}$$

schemes will rely on the fact that the updates in (20) are subgradients of the dual function of (9), see e.g. [19, Ch. 6]. Employing unbiased stochastic subgradients of the dual function for allocating resources in wireless *fading* networks has received attention in recent years [15]–[17]. The focus here is on algorithms that operate also over *static* channels. To be specific, assuming that the updates in (20) are bounded, the following result guarantees feasibility and near optimality of the online resource allocation.

Proposition 1: *As $N \to \infty$, it holds that: (i) the average constraints* (2)*,* (3)*,* (4)*,* (6) *are satisfied with equality and $\frac{1}{N}\sum_{n=1}^{N}\bar{a}_m^*(\hat{\boldsymbol{\lambda}}[n]) \geq \check{a}_m$; and (ii) with $\delta_{\mathsf{P}}(\mu)$ denoting a positive number proportional to μ, the achieved objective satisfies [cf.* (9)*] $\sum_{m=1}^{M} U_m\left(\frac{1}{N}\sum_{n=1}^{N}\bar{a}_m^*(\hat{\boldsymbol{\lambda}}[n])\right) \geq \mathsf{P}^* - \delta_{\mathsf{P}}(\mu)$.*

In words, Proposition 1 guarantees asymptotic optimality of the online primal variables because they are feasible and achieve a value (performance) arbitrarily close to P^*, which is the optimal value of the original solution of (9). Proposition 1 can be proved using the results on the convergence of averages of subgradient methods [20]. If optimality is the main concern, the estimates in (20) can be updated using a diminishing stepsize satisfying $\lim_{N\to\infty}\sum_{n=1}^{N}\mu[n] = \infty$ and $\lim_{N\to\infty}\sum_{n=1}^{N}(\mu[n])^2 < \infty$. In such a case, there is no loss of optimality, but the tracking capabilities of the schemes are sacrificed.

Remark 1: Online estimates are well suited for computing average multipliers, but not for the instantaneous multipliers because the corresponding constraints need to hold for each instant and Proposition 1 guarantees optimality only in the long term (i.e., after averaging over time).

V. Real-time users

The utility functions in (9a) and the minimum rate constraints in (5) considered so far guarantee QoS for the *long-term* arrival rate (elastic traffic). The changes required to offer guarantees on the *short-term* rate (real-time traffic) are the aim of this section. Let us remind that $\mathcal{M}_{III}$ is formed by users with real-time traffic. Hence, they will have to transmit the packets received from the higher layers to the BS instantaneously and, if they decide to send their traffic through an RS, the RS has to forward the packets instantaneously. The real-time guarantee will then amount to ensure that the instantaneous (short-term) rate between user $m \in \mathcal{M}_{III}$ and the BS exceeds a minimum pre-specified level $\check{a}_m$ –if needed, $\check{a}_m$ can be rendered different for each n.

There are different approaches to deal with real-time traffic [14]. One is to consider that the traffic can tolerate a small delay D, so that real-time traffic received at instant n has to be transmitted before instant $n+D$. This problem can be solved exactly with exponential complexity by using dynamic programming; see, e.g., [22]; or approximately using time-varying priority weights, see e.g., [23]. A different approach is to give real-time traffic maximum priority, so that the required rate $\check{a}_m$ is guaranteed for each at every instant n [13], [24]. In this paper we will follow the second approach, so that the solution is optimal and the complexity is kept under control. Moreover, we have to clarify that the instantaneous rate $\check{a}_m$ will not be guaranteed within one slot, but within two slots (i.e, rate averaged across n and $n+1$, with n odd). We do this to allow real-time traffic to be routed through the RS (remind that relayed packets need two slots to reach the BS). Hence, to account for real-time users in our formulation, the following instantaneous constraint will be incorporated into the optimization problem [cf. (4) and (5)]

$$\frac{1}{2}\sum_{j=0}^{1}\sum_{k=1}^{K}\sum_{l=0}^{L} r_{m,k}^{l,0}[n+j]w_{m,k}^{l,0}[n+j] \geq \check{a}_m/(1-\beta), \quad m \in \mathcal{M}_{III},\ \forall \text{ odd } n. \tag{21}$$

In addition to this, one has to guarantee that the real-time packets received by the RS are relayed to the BS within the next slot. This implies that constraints (7) and (8) need to be enforced also for users in $m \in \mathcal{M}_{III}$. Finally, since the rate requirements for real-time users are instantaneous (and not average), there is no need to define a utility function for their average rate [25].

For the schemes in Sec. IV to account for real-time traffic, constraints (21) need to be incorporated into (9) and hence, into the instantaneous optimization problem in (15), which now yields also the optimum scheduling, power and rate variables for users in $\mathcal{M}_{III}$. For this purpose, it is important to stress that all links (m,l,b) involving real-time users are elements of set $\mathcal{T}_2$ [cf. (15)]. The approach to guarantee the constraints (21) will be similar to that used for (7). Specifically, the constraints will be dualized and the *optimum* value of the corresponding multiplier will be found for each time instant. Let $\rho_m^{0,0}[n]$ denote the multiplier associated with (21) and $\rho_m^{0,0*}[n]$ its optimum value. The values of $p_{m,k}^{l,b*}(n,\boldsymbol{\lambda})$, $r_{m,k}^{l,b*}(n,\ \boldsymbol{\lambda})$ for real time users will be found after substituting

$\rho_m^{0,0} = \rho_m^{0,0*}[n]$ into (16) and (13), respectively. Similarly, the optimum value for the scheduling coefficients is found after incorporating real-time users and links into (18) and replacing $\rho_m^{0,0}$ with $\rho_m^{0,0*}[n]$ in the link indicators of such users. The value of $\rho_m^{0,0*}[n]$ can be found iteratively using [cf. (19)]

$$\rho_m^{0,0}[n,i+1] := \left[\rho_m^{0,0}[n,i] + \mu[i]\Big(\check{a}_m/(1-\beta) - \sum_{l=0}^{L} \tilde{r}_m^{l,0*}[n,i]\Big)\right]_0^{\infty} \tag{22}$$

The iterations in (22) need to be carried out jointly (simultaneously) with those in (19). As mentioned earlier, the formulation can easily accommodate real-time requirements that vary with time. To do so, one only needs to replace $\check{a}_m$ with $\check{a}_m[n]$ in (22). Note also that very high requirements (especially for channels with severe fading) may render the problem infeasible, so that $\rho_m^{0,0}[n,i+1]$ grows unboundedly. The discussion on admission control mechanisms after (9) also applies here.

Finally, since no new average constraints (multipliers) need to be introduced, the online estimates in (20) and the results in Proposition 1 hold true also when real-time users are present.

For clarify, the operation of the algorithm developed in Secs. V and IV is summarized in Fig. 1.

VI. Extending the results to different setups

The focus in this section is on briefly describing extensions that can be easily handled and do not entail significant changes to the schemes developed so far. For each extension, the main changes in the formulation are identified, and pertinent citations are provided.

A. Downlink setup

The first step to address the optimal design for the downlink setup is to reinterpret some of the notation. Specifically, the role of users as transmitters and the BS as receiver is reversed. This way, $h_{m,k}^{0,0}$ represents now the (power) gain in the kth channel when the BS transmits to the mth user, and $\bar{a}_m$ the average exogenous rate at the BS destined for user m.

Secondly, the M individual power constraints in (2) must be replaced with the single constraint

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{l=0}^{L} p_{m,k}^{l,0}[n] w_{m,k}^{l,0}[n] \leq \check{p}^B, \tag{23}$$

where $\check{p}^B$ denotes the maximum average transmit-power for the BS. This in turn implies that instead of M multipliers $\{\pi_m^{0,0}\}_{m=1}^M$, now only a single multiplier $\pi^{0,0}$ needs to be considered. Clearly, $\{\pi_m^{0,0}\}_{m=1}^M$ were associated with the M constraints in (2), while $\pi^{0,0}$ is associated with the single constraint in (23). After replacing $\pi_m^{0,0}$ with $\pi^{0,0}$, the expressions for the optimal $\bar{a}_m^*(\boldsymbol{\lambda})$, $p_{m,k}^{l,b*}(n,\boldsymbol{\lambda})$, $r_{m,k}^{l,b*}(n,\boldsymbol{\lambda})$, and $w_{m,k}^{l,b*}(n,\boldsymbol{\lambda})$ presented in Secs. IV and V also hold for a downlink setup.

Finally, the M online updates in (20a) need to be replaced with the single update

$$\hat{\pi}^{0,0}[n+2] := \left[\hat{\pi}^{0,0}[n] - \mu\left(\check{p}^B - \sum_{m=1}^{M}\sum_{l=0}^{L} \tilde{p}_m^{l,0}(n,\hat{\boldsymbol{\lambda}}[n])\right)\right]_0^{\infty} \tag{24}$$

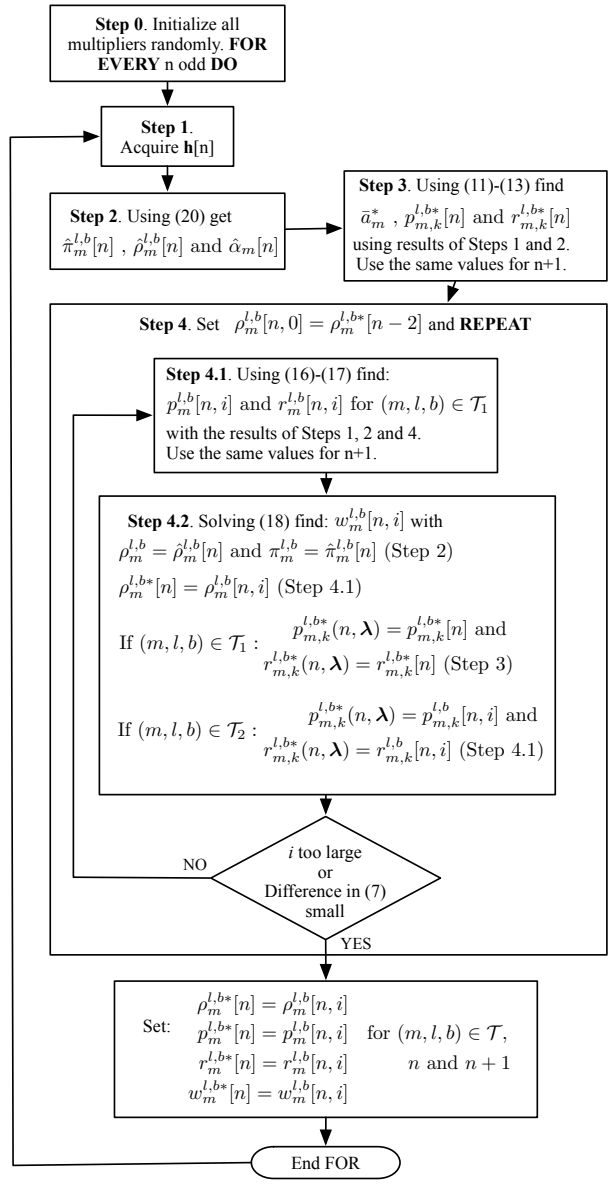

Fig. 1: Pseudo-code of the proposed algorithm.

B. Adaptive modulation and code modes

Terminals in practical wireless communication networks are not able to transmit with an arbitrary rate, rather only a finite set of Adaptive Modulation and Coding (AMC) modes can be implemented. Let $Q_{m,k}^{l,b}$ be the number of AMC modes user m can implement in hop (l,b) and channel k in most real systems, the *set* of available modes will be the same for every user, hop and channel, so that $Q_{m,k}^{l,b} = Q$). Moreover, let $r_{m,k,q}^{l,b}$ be the transmit-rate that is achieved when the qth mode (with $1 \leq q \leq Q_{m,k}^{l,b}$) is implemented. Clearly, if the channel power gain $h_{m,k}^{l,b}$ and the BER requirement ϵ_m are given, every $r_{m,k,q}^{l,b}$ gives rise to its corresponding $p_{m,k,q}^{l,b}$.

With this operating conditions, the rate-power function $C_{m,k}^{l,b}(\mathbf{h},\cdot)$ in Sec. II is not continuous any more, and the optimal resource allocation at the physical layer consists in selecting the $(p_{m,k,q}^{l,b}, r_{m,k,q}^{l,b})$ pair for each active link.

Using a similar approach than that in [15], it can be shown that the optimal rate and power allocation for this case are $r^{l,b*}_{m,k}(n,\hat{\boldsymbol{\lambda}}[n]) = r_{m,k,q^*}$ and $p^{l,b*}_{m,k}(n,\hat{\boldsymbol{\lambda}}[n]) = p_{m,k,q^*}$, where

$$q^* = q^*(m,k,l,b,n,\hat{\boldsymbol{\lambda}}[n]) := \arg\max_q \left\{\varphi(\rho^{l,b}_m[n],\hat{\pi}^{l,b}_m[n],r^{l,b}_{m,k,q},p^{l,b}_{m,k,q})\right\}_{q=1}^{Q^{l,b}_{m,k}} \quad (25)$$

C. Time-varying fading channels

Static channels have been considered so far because: i) many relay systems are deployed in scenarios where that assumption is accurate (terminals are fixed and there exists direct line of sight); and ii) the notation and mathematical derivations were a bit simpler than those for the time-varying case. However, our schemes can be easily modified to account for time-varying fading channels, so that $\mathbf{h}$ becomes a (time-varying) random process $\mathbf{h}[n]$. This entails that some of the optimality claims need to be reinterpreted. A significant difference is that for the scheduling derived in Secs. IV and V being optimal we need $\mathbf{h}[n] \approx \mathbf{h}[n+1]$. Another difference is that the optimality and feasibility claims in Proposition 1 need to be restated in a probabilistic sense. If $\mathbf{h}[n]$ is random, the updates in (20) are not subgradients of the dual function any more. Rather, they are stochastic subgradients (random variables whose expected value is the value of the true subgradient). The specific changes in the claims will depend on the time correlation model assumed for the channel; see, e.g., [17] for i.i.d. case.

D. Other relay configurations

Our design methodology can also be applied when the RSs have to relay the packets in the same channel (frequency band) where they were received, as it is the case when channels correspond to OFDM subcarriers. In such a case, (7) needs to be replaced with $r^{l,0}_{m,k}[n]w^{l,0}_{m,k}[n] \leq r^{l,1}_{m,k}[n+1]w^{l,1}_{m,k}[n+1]$. Compared to the original algorithm, the resulting algorithm will achieve a lower utility value. However, it will decrease the computational complexity (especially, for time-varying channels). Other systems may prevent real-time traffic from being routed through the RS. In such a case, it would suffice to ignore (7) for $m \in \mathcal{M}_{III}$ and drop the second term in the left hand side of (21).

VII. Numerical results

In this section, several experiments are carried out to investigate the performance of the proposed algorithms. The default test-case is the uplink of an OFDMA system operating over a frequency selective Rayleigh channel. To simulate a challenging scenario with high frequency selectivity, the number of paths is set to 8, all with the same power, and $K = 18$ channels (groups of subcarriers) are considered. The simulations are run during $N = 4000$ time instants (twice for the first test case because the rate requirements are modified after the first 4000 instants). Parameter β in (4) is set to zero. The default number of RSs is two (unless other number is explicitly mentioned) and they are deployed so that their transmitting conditions (channel gains and transmit powers) are better that those for the regular users.

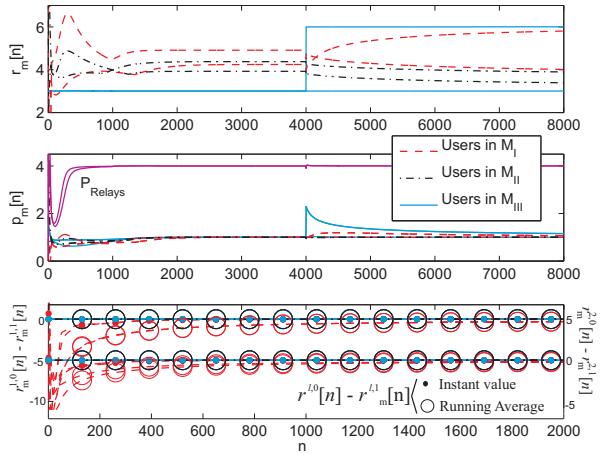

Fig. 2: Primal variables for Test Case 1.

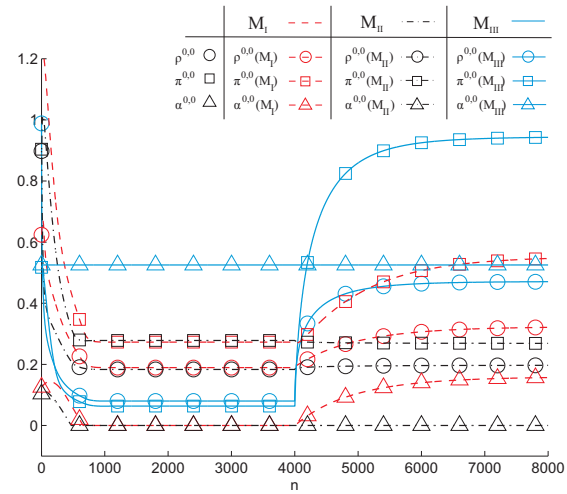

Fig. 3: Lagrange multipliers for Test Case 1. Only one user per subset $\mathcal{M}$ is represented.

Test Case 1: feasibility and convergence. Six users (two in $\mathcal{M}_I$, two in $\mathcal{M}_{II}$ and two in $\mathcal{M}_{III}$) are considered. Some users are placed near the BS and the rest of them near one of the RSs, as shown in Table I. Additionally, to confirm that our algorithms are able to track changes in the setup, the minimum rate requirements are modified online. Specifically, for the first $N/2$ slots, the minimum rates requirements are 3 for every user, and then two of the users (one in $\mathcal{M}_I$ –call it m_1– and the other in $\mathcal{M}_{III}$ –call it m_3–) increase their rate requirement up to 6. Simulated parameters are listed in Table I.

Fig. 2 represents the trajectories of primal variables. The upper subplot represents the instantaneous rate transmitted by each user (i.e., $r_m[n] := \sum_{l=0}^{L} \tilde{r}^{l,0}_m[n]$); the middle subplot the instantaneous transmit power for the users and the relays; in the lower subplot the violation of the instantaneous flow conservation constraint at both RSs (i.e., $\tilde{r}^{l,0}_m[n] - \tilde{r}^{l,1}_m[n]$) is represented with solid small circles and its running average (i.e., $\frac{1}{n+1}\sum_{v=0}^{n} \tilde{r}^{l,0}_m[v] - \tilde{r}^{l,1}_m[v]$) with big circles. For clarity purposes, in this subplot the lines corresponding with both RSs have been differentiated with different scales represented

TABLE I: PARAMETERS FOR THE TEST CASES (TC). THE UTILITY FUNCTION IS SET TO $U_m(x) = \log(0.1 + x)$ FOR ALL TCS.

Test Case	Channel	$(\bar{h}_m^{0,0}, \bar{h}_m^{1,0}, \bar{h}_m^{1,1}, \bar{h}_m^{2,0}, \bar{h}_m^{2,1})$	$\check{a}_m, m \in \mathcal{M}_I$	$\check{a}_m, m \in \mathcal{M}_{II}$	$\check{a}_m, m \in \mathcal{M}_{III}$	$\check{p}_m, \forall m$	$\check{p}^R$
TC1	Constant	$m \in \{1,5\}$: (7, 7, 4, 7, 7) $m \in \{2,6\}$: (0, 10, 4, 7, 7) $m = 3$: (7, 4, 7, 7, 7) $m = 4$: (0, 4, 10, 7, 7)	$n < \frac{N}{2}$: [3,3] $n \geq \frac{N}{2}$: [6, 3]	$n < \frac{N}{2}$:[3, 3] $n \geq \frac{N}{2}$:[3, 3]	$n < \frac{N}{2}$:[3, 3] $n \geq \frac{N}{2}$:[6, 3]	1.5	4
TC2	Fading	The same as for TC1	[3, 3]	[3, 3]	[3, 3]	1.5	4
TC3	Constant	$m \in \{1,5\}$: (7, 7, 4, 9, 9) $m \in \{2,6\}$: (0, 10, 4, 9, 9) $m = 3$: (7, 4, 7, 9, 9) $m = 4$: (0, 4, 10, 9, 9)	[3, 3, 3]	[3, 3, 3]		1	4
TC4	Constant	$m = 1$: (0, 7, 6, 9, 9) $m = 2$: (3, 5, 9, 9, 9) $m = 3$: (0, 8, 6, 9, 9) $m = 4$: (3, 5, 5, 9, 9) $m = 5$: (0, 9, 6, 9, 9) $m = 6$: (0, 6, 9, 9, 9)	[3.5, 3.5]	[3.5, 3.5]	[3.5, 3.5]	1	4

at both sides of the figure. Moreover, only the first 2000 time instants are represented to simplify the observation of the results. The first observation is that our algorithm converges to a feasible solution. Specifically, all maximum power constraints are satisfied with equality –see middle subplot–, and all minimum rate constraints are satisfied too –see upper subplot–. We also observe that real-time users in $\mathcal{M}_{III}$ get the required rate *for each and every* time instant, while elastic users only get their minimum rate after averaging across time. Moreover, the upper subplot confirms that the rate of user m_3 changes instantaneously when its minimum rate requirement is modified. Middle subplot confirms that our algorithm is also able to drive the average rate at which m_1 transmits from 4 to 8, but in this case the rate changes gradually (taking advantage of the fact of m_1 transmitting an elastic flow). During a short interval after $n = 4000$, user m_3 transmits with a very high power (the algorithm reacts aggressively to guarantee the satisfaction of the real-time rate requirement). This behavior contrasts with the smooth evolution of the power transmitted by user m_1. Regarding the flow conservation constraints, the lower subplot confirms that users in $\mathcal{M}_{II}$ and $\mathcal{M}_{III}$ satisfy the constraint instantaneously, while users in $\mathcal{M}_I$ violate the instantaneous constraint, but satisfy the average one.

Fig. 3 depicts the value of the multipliers at every time instant. To reduce the number of lines, only the multipliers associated with three users (one per set) are shown. The main observation is that indeed the multipliers associated with constraints that are active are non-zero. Also, the trajectories of the multipliers illustrate that after instant $n = 4000$, the value of the multipliers associated with the QoS constraints of users m_1 and m_3 change to account for the new requirements (for example, $\alpha_{m_1}[n] > 0$ after $n > 4000$).

Test Case 2: fading channels. In this experiment we illustrate the dynamic behavior of our schemes by investigating the effect of time-varying fading (the correlation follows a Jakes model with coherence time of $T_{coh} = 100$ [18]). The second row of Table I lists the main parameters for this test case and Figs. 4 and 5 are the counterparts of Figs. 2 and 3.

A striking difference is that due to channel variability, now

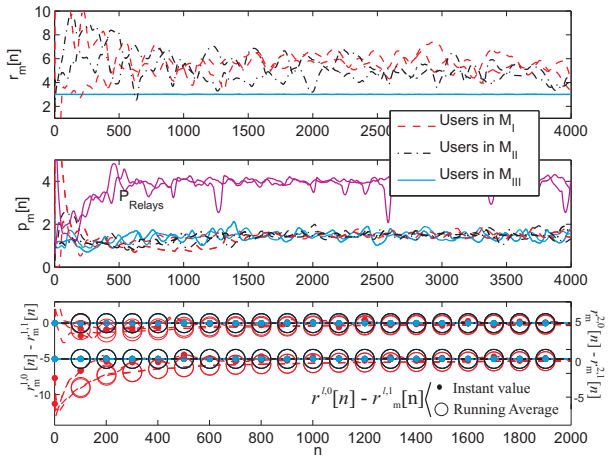

Fig. 4: Primal variables for Test Case 2.

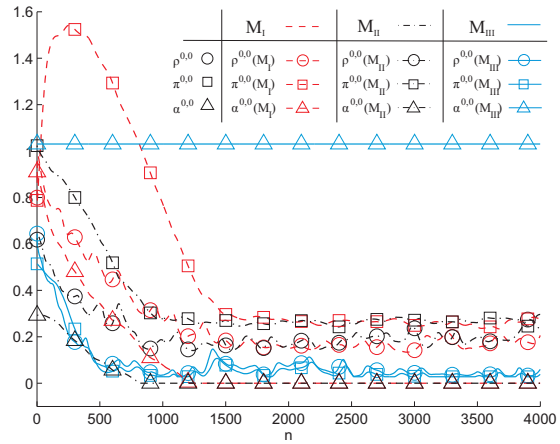

Fig. 5: Lagrange multipliers for Test Case 2.

the instantaneous variables (both primal and dual) vary sharply with time. Despite of time-variability, Fig. 4 illustrates that all the considered constraints are satisfied. Namely: i) average rate, power and flow conservation constraints are satisfied in the long-term and ii) minimum transmit rate for users in $\mathcal{M}_{III}$

and instantaneous flow conservation rates at the RSs for users in $\mathcal{M}_{II}$ and $\mathcal{M}_{III}$ are satisfied for each and every time instant (recall that the multipliers for ii) are optimally found for every n). Note however, that minimum rate constraints for users in $\mathcal{M}_{III}$ could have been violated if the fading had been too severe; see discussion after (22). A consistent behavior is observed in Fig. 5. All rate multipliers keep varying with time. The instantaneous multipliers vary because the optimum value $\boldsymbol{\lambda}^*[n]$ depends on the specific channel realization and thus it is different for each instant. The average multipliers vary because the estimates $\hat{\boldsymbol{\lambda}}[n]$ never converge to the optimum static value $\boldsymbol{\lambda}^*$, but hover around it.

Test Case 3: optimality without real-time traffic. In test cases 3 and 4 we compare our algorithms with other (suboptimal) alternatives. First we focus on a scenario without real-time users and compare: a) our optimal algorithm (referred to as A1-NRT); b) a scheme (call it A2) with fixed power and channel allocation (M/K consecutive channels for each user) which optimally adapts the rate and the hop to transmit (directly to the BS or through the RS); and c) a scheme (call it A3) like A2 but where the power loadings are also optimal. The third row in Table I shows the main parameters for this test case and Table II the obtained results. Not only A1-NRT does outperform the other alternatives (both in terms of sum-utility and sum-rate), but it gives rise to a feasible solution (both A2 and A3 fail to fulfills some of the rate constraints). As expected, A3 outperforms A2.

Test Case 4: optimality with real-time traffic. In this experiment users with real-time requirements are considered (see fourth row in Table I for further details). In this case our optimal algorithm (referred to as A1-RT) is compared to a suboptimal algorithm (call it A4). A4 is similar to A3 but it gives maximum priority to real-time users, so that they are able to keep their best channels to fulfill their instantaneous rate constraint. For this test, the users are sparsely distributed around the BS and the RSs, as shown in Table I. The results of the experiment are listed in the fourth and fifth row of Table II, confirming that A1-RT outperforms A4 in terms of sum rate and utility. Note that the difference of performance for both algorithms strongly depends on the scenario (being always better A1-RT) and that the harder the scenario is (higher data rates requirements and lower SNRs) the higher the difference in performance. Finally, it is worth remarking that A1-RT exhibits lower performance than that of A1-NRT. Although the scenarios are not identical, they are quite similar, and the difference is mainly due to the real-time constraints, which are more restrictive. To corroborate this we simulate the performance of our algorithm in two additional setups: one where all users belong to $\mathcal{M}_{III}$ and another one where all users belong to $\mathcal{M}_{I}$. The results are listed in the two last rows of of Table II.

VIII. Concluding summary

Cross-layer algorithms were designed to allocate resources in a cellular system equipped with several relay stations that allow for two-hop transmissions. The optimal allocation of resources consisted of: i) the (transport) rate that can be accepted from higher layers, ii) the power and (physical) rate that each terminal has to use, iii) the terminals which can transmit on each of the channels, and iv) whether the users should transmit directly to the base station or use a relay station. The algorithms were designed using convex optimization and dual decomposition tools. The main challenge was the joint consideration of long-term (elastic) and short-term (real-time) constraints. The latter were used to: a) guarantee a minimum instantaneous rate for real-time traffic and b) enforce instantaneous forwarding at the relay stations. It turned out that the developed resource allocation strategy depended on the channel conditions, user-specific Lagrange multipliers (with are associated with the users' QoS requirements), and the operating conditions of the specific setup. The variables (resources and multipliers) associated with the real-time and elastic requirements were found separately. A classical iterative method was used to calculate the multipliers for the instantaneous constraints, while a low-complexity approximate method was developed to estimate online the multipliers for the long-term constraints.

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TABLE II: PERFORMANCE OF THE PROPOSED ALGORITHM AND COMPARISON WITH SIMPLER APPROACHES. THE VALUES ARE FOUND AFTER AVERAGING ACROSS 4000 TIME INSTANTS.

Alg.	$[\overline{a}_m, \forall m]$	$[\overline{p}_m, \forall m]$	$\sum_m \overline{a}_m$	$\sum_m U_m(\overline{a}_m)$
A1-NRT	[4.7, 7.0, 4.7, 6.1, 5.3, 4.9]	[1.0, 1.0, 1.0, 1.0, 0.9, 0.9]	32.7	10.2
A2	[4.8, 4.1, 4.8, 2.2, 3.8, 1.7]	[1.0, 1.0, 1.0, 1.0, 1.0, 1.0]	21.4	7.4
A3	[4.5, 5.3, 4.4, 3.8, 4.1, 2.6]	[1.0, 1.0, 1.0, 1.0, 0.9, 1.0]	24.6	8.5
A1-RT	[5.3, 6.0, 5.2, 4.8, 3.5, 3.5]	[0.9, 1.0, 0.9, 1.0, 1.0, 1.0]	28.4	9.3
A4	[4.3, 4.7, 4.6, 4.3, 3.5, 3.5]	[1.0, 0.9, 0.9, 1.0, 1.0, 1.0]	24.9	8.6
A1-NRT all in $\mathcal{M}_I$	[4.8, 6.1, 5.4, 3.8, 6.1, 4.6]	[1.0, 1.0, 1.0, 1.0, 1.0, 1.0]	30.8	9.9
A1-RT all in $\mathcal{M}_{III}$	[3.5, 3.5, 3.5, 3.5, 3.5, 3.5]	[0.9, 1.0, 1.0, 0.9, 1.0, 0.9]	21.0	7.7

TABLE III: SUMMARY OF MOST SIGNIFICANT NOTATION.

Symbol	**Meaning**
n	Time slot index
$M \mid L \mid K$	Number of users | Number of RSs | Number of channels
$m \mid l \mid k$	User index | RS index | Channel index
b	Binary variable, $b = 0$ if the origin is a user and $b = 1$ if the origin is an RS
$\mathcal{S} \mid (l, b)$	Set of available hops: {User to BS, User to RS, RS to BS} | Hop identifier
$\mathcal{M}_I \mid \mathcal{M}_{II} \mid \mathcal{M}_{III}$	Users w/ non real-time traffic (RT) & *asynchronous relaying* | non RT & *instantaneous relaying* | RT
$\mathcal{T} \mid (m, l, b)$	Set of all possible links | Link identifier for user m and hop (l, b)
$\mathcal{T}_1 \mid \mathcal{T}_2$	Links with *average* | *instantaneous* constraints
$\mathbf{h} \mid h_{m,k}^{l,b}$	Channel gain vector | Channel gain for user m on channel k for hop (l, b)
$w_{m,k}^{l,b}$	Scheduling variable (fraction of time user m occupies channel k for hop (l, b))
$p_{m,k}^{l,b}[n] \mid r_{m,k}^{l,b}[n]$	*Power* | *Rate* transmitted by user m over channel k for hop (l, b) during time slot n
$C_{m,k}^{l,b}(\mathbf{h}, p_{m,k}^{l,b}[n])$	Capacity (rate-power) function for user m over channel k and hop (l, b)
$\check{p}_m^U \mid \check{p}_l^R \mid \check{p}^B$	Maximum average power consumed by user m | RS l | *base station*
$\check{p}^k$	Maximum peak (instantaneous) power on channel k
$\bar{a}_m \mid \check{a}_m$	*Average* | *minimum* arrival rate of user m
$U_m(\bar{a}_m)$	Utility function of user m's service rate
P^*	Sum-utility achieved by the optimal resource allocation [cf. (9)]
$\pi_m^{l,b}$	Lagrange multiplier for average power constraint (m, l, b)
$\rho_m^{l,b} \mid \rho_m^{l,b}[n]$	Lagrange multiplier for *average* | *instantaneous* flow-conservation constraint (m, l, b)
α_m	Lagrange multiplier for average arrival rate constraint m
$\boldsymbol{\lambda}$	Vector gathering all Lagrange multipliers
x^*	Optimum value for a given variable x
$\varphi(\rho_m^{l,b}, \pi_m^{l,b}, r_{m,k}^{l,b}[n], p_{m,k}^{l,b}[n])$	Link-quality indicator
$\tilde{p}_m^{l,b*} \mid \tilde{r}_m^{l,b*}$	Effective *power* | *rate* for user m and hop (l, b)
$\rho_m^{l,b}[n, i] \mid \tilde{r}_m^{l,b*}[n, i]$	Estimate of $\rho_m^{l,b}[n] \mid \tilde{r}_m^{l,b*}[n]$ at iteration i
$\hat{\pi}_m^{l,b}[n]$, $\hat{\rho}_m^{l,b}[n]$, $\hat{\alpha}_m[n]$	Stochastic estimate of $\pi_m^{l,b*}$, $\rho_m^{l,b*}$, α_m^* at slot n
μ	Stepsize

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